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RESEARCH IN DIGITAL ADAPTIVE FLIGHT CONTROLLERS

Howard Kaufman

Prepared by
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for Langley Research Center



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16. Abstract A design study of adaptive control logic suitable for implementation in modern airborne digital flight computers has been conducted. Both explicit controllers which directly utilize parameter identification and implicit controllers which do not require identification were considered. Extensive analytical and simulation efforts resulted in the recommendation of two explicit digital adaptive flight controllers. These interface weighted least squares estimation procedures with either control logic developed using optimal regulator theory or with control logic based upon single stage performance indices.					
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FOREWORD

The investigation described in this report was performed by the Electrical and Systems Engineering Department of Rensselaer Polytechnic Institute for the Flight Dynamics and Control Division of the Langley Research Center as a part of the Digital Fly-By-Wire Program. It was carried out during the period September 15, 1972 - June 15, 1975. The investigation was headed by Professor Howard Kaufman who was assisted by three graduate students supported under the grant.

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SYMBOLS

A_m	Model system matrix
A_p	Plant system matrix
B_m	Model input matrix
B_p	Plant input matrix
\underline{e}	Error vector, $\underline{x}_p - \underline{x}_m$
F, F_p	Continuous system matrices
G, G_p	Continuous input matrices
H	Measurement selector matrix
$I (I_p)$	Identity matrix (of dimension $p \times p$)
J	Performance index
K	Feedback gain matrix (eq. 3.6 or eq. 3.19)
$K(k), K_q(k), K_x(k)$	Estimator correction gains
K_{u_m}	Gain matrix multiplying pilot input \underline{u}_m
K_{u_p}	Gain matrix multiplying plant input \underline{u}_p
K_{x_p}	Gain matrix multiplying plant state \underline{x}_p
K_{x_m}	Gain matrix multiplying model state \underline{x}_m
n	Number of plant or model states
$\underline{n}(k)$	Measurement noise vector at sample number k
P	Riccati matrix or covariance of estimate
p	Incremental roll rate
Q	Weighting matrix
q	Incremental pitch rate
\underline{q}	Vector of parameters to be identified

\hat{q}	Estimate of q
R	Weighting matrix or observation noise covariance
r	Incremental yaw rate
T	Sample period
u_m	Pilot input vector
u_p	Incremental aircraft input vector = $u_p^a - u_p^t$
u_p^a	Actual control input vector
u_p^t	Trim control vector
v	Incremental velocity
v_p	Per sample change in the aircraft control u_p
W	Covariance of fictitious parameter noise
x_m	Model state vector
x_p	Incremental plant state vector = $x_p^a - x_p^t$
x_p^a	Actual plant state vector
x_p^t	Trim state vector
\hat{x}_p	Estimate of x_p
y_p	Aircraft measurement vector
α	Incremental angle of attack
β	Incremental Sideslip angle
δ_a	Incremental aileron deflection
δ_e	Incremental elevator deflection
δ_r	Incremental rudder deflection
δ_T	Incremental thrust

ϕ	Incremental roll angle
θ	Incremental pitch angle

RESEARCH IN DIGITAL ADAPTIVE FLIGHT CONTROLLERS

by

Howard Kaufman*

SUMMARY

Adaptive flight control systems are of interest because of their potential for providing uniform stability and handling qualities over a wide flight envelope despite uncertainties in the open loop characteristics of the aircraft. Since such controllers combine the functions of performance assessment, state and parameter estimation, gain adjustment, and control computation, it is most advantageous to consider implementation in digital rather than analog fly-by-wire systems.

To this effect, a study has been made in order to define an implementable digital adaptive control system which can be used for a typical fighter aircraft. Since online adjustment of the control gains requires an easily computable index of performance, a model of the desired aircraft behavior was developed and used in real time for computing the error between plant and model output.

With regard to control gain adjustment, based upon the error in model following, consideration was primarily given to explicit adaptive control systems which directly utilize parameter estimates for gain adjustments. Implicit adaptive controllers, while not requiring the explicit computation of parameter estimates, are not readily tunable to system specifications unless the plant and model structures are such that a "perfect model following" control law can be defined.

Relative to the need for both parameter and state estimates, it was observed that online estimation of the states and parameters is best performed by a weighted least squares procedure which first identifies the parameters directly from the noisy measurements, and then computes state estimates using both the parameter estimates and the noisy measurements.

Based upon analytical studies and simulation of both the linear and nonlinear aircraft equations of motion, two explicit adaptive controllers were recognized as being suitable for inflight implementation. The first design uses control and gain adaptation logic developed using infinite time linear optimal regulator theory. Control gains are therefore stabilizing (for a fixed system) and are adjusted in response to parameter changes through

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an iterative correction made to the Riccati matrix. The second design is based upon single stage performance indices which result in gains that are immediately computable by formula evaluation. To assure stability of the closed-loop system, a simplified Riccati iteration was used for correcting the feedback gain matrix.

Results showed that these adaptive controllers were effective in compensating for parameter variations and were even capable of rapid recovery from highly erroneous parameter estimates which could in fact define a set of destabilizing gains.

1. INTRODUCTION

1.1 Background

Fly-by-wire flight control systems have been of considerable interest to designers because of their advantages over mechanical linkages in coping with the complex control problems associated with high performance aircraft and space vehicles.^{1,2} Furthermore, with the present capabilities for incorporating integrated circuits into lightweight low cost minicomputers and microcomputers, digital implementation of fly-by-wire control becomes especially attractive. Digital logic is itself very reliable and with adequate redundancy incorporated into the design, such a system can be designed to insure adequate flight safety.³

Another feature of digital implementation which makes it extremely advantageous is the potential for the implementation of complex control systems which incorporate high order nonlinearities and which utilize time sharing for multiple loop control. One such complex control structure is an adaptive system which is capable of online adjustment of the control parameters in response to changing flight characteristics. The desirability for such adaptive control systems has been established for providing uniform stability and handling qualities over the complete flight envelope despite drastic changes in the aircraft's open loop characteristics, even in the presence of disturbances.^{4,5} Although control parameters (e.g., gains) can be scheduled as a function of altitude and mach number, this is not always desirable because of the inaccuracies which arise from computing offline and then scheduling the aircraft characterizations at different flight conditions and the effects of unpredictable uncertainties themselves (e.g., sloshing, fuel weight, structural changes).

Thus, because of the attractiveness of adaptive control and the availability of digital flight computers, the possibility of implementing a digital adaptive flight control system has been studied since September 1972. Preliminary results of this study have led to two adaptive control algorithms, namely:

- . An interfacing of linear optimal regulator logic with a weighted least squares identifier.
- . An interfacing of control logic designed using a single stage performance index with a weighted least squares identifier.

Although implementation of this latter structure was very simple relative to that of the former, stability could not be guaranteed even if identification were perfect; thus some modification was necessary. Another important result from this study was the recommendation that in view of observed convergence properties, parameter identification be performed separately from state estimation.

Towards the goal of developing digital adaptive flight control algorithms suitable for implementations in an onboard digital computer, continued efforts have been concerned with stabilizing the single stage adaptive control logic, determining identification requirements and evaluating the designed control systems on NASA Langley's nonlinear six degree of freedom simulation.

It should be noted that since stability in the presence of large parameter variations was of concern, initial studies were concerned with explicit adaptive controllers which make direct use of online parameter identification.^{6,7} Implicit adaptation procedures which do not require explicit parameter estimates for adjusting controller gains, do not guarantee stability unless certain idealistic structural constraints hold.^{8,9} However, because of the attractiveness of eliminating the need for identification logic, some effort has since been expended in defining a stable implicit adaptive controller useful for digital flight control.

1.2 Objectives

In view of the desirability for designing an implementable digital adaptive flight control system, Rensselaer Polytechnic Institute has since September 1972, performed research related to the following overall objectives:

- 1) Develop and evaluate adaptive control logic using the linearized lateral and longitudinal equations of motion for a typical fighter aircraft.
- 2) Evaluate the linearized designs on NASA Langley's nonlinear six degree of freedom simulation.
- 3) Recommend digital adaptive control logic suitable for inflight implementation

1.3 Scope and Outline

Development of a digital adaptive flight control system requires the:

- . Development of mathematical models
- . Formulation of the controller structure and the gain adjustment procedures
- . Design of parameter and state estimation logic

As stated in the preceding sections, all initial designs were performed using the linearized lateral and longitudinal equation for a typical fighter aircraft. The precise modelling of these dynamics along with a description of the representations of the actuator dynamics, sensors, and bending modes is contained in Section 2.0.

Using these models, two distinct explicit adaptive controllers were designed based upon optimal linear regulator theory and a single stage index modified so as to insure stability. These are discussed in Section 3.1.1. Results of a parallel study on the feasibility of implicit adaptive controllers are presented in Section 3.1.2.

In view of the need for parameter estimation for explicit adaptive control, an analytical and experimental study was made of candidate procedures for online identification. These results are contained in Section 3.2.

Results of evaluating the overall adaptive control system on both the linearized and nonlinear equations of motion are presented in Section 4, and finally, conclusions regarding implementation are presented in Section 5.

1.4 Significance

Development of a digital adaptive flight control system is of significance not only to the particular aircraft considered but also to digital process control in general. Such a development represents an important application of modern digital control theory that is a step towards the narrowing of the gap between theory and practice.

Of immediate significance was the demonstration of two digital adaptive control flight controllers which are capable of identifying and compensating for time varying uncertainties in the open loop characteristics without the need for altitude and mach number information.

2. PROBLEM FORMULATION

2.1 Mathematical Representations

2.1.1 Aircraft Dynamics

2.1.1.1 Linear Representation

The linearized dynamics of the aircraft as supplied by NASA can be represented by the vector state equation

$$\dot{\underline{x}}_p = F_p \underline{x}_p + G_p \underline{u}_p \quad (2.1)$$

where \underline{x}_p denotes the incremental state vector

\underline{u}_p denotes the incremental control vector

and F_p and G_p are matrices of the appropriate dimension.

In particular, for linearized lateral motion:

$$\underline{x}_p = \begin{pmatrix} p \\ r \\ \beta \\ \phi \end{pmatrix} = \begin{pmatrix} \text{roll rate} \\ \text{yaw rate} \\ \text{sideslip angle} \\ \text{roll angle} \end{pmatrix} \quad (2.2)$$

and $\underline{u}_p = \begin{pmatrix} \delta_a \\ \delta_r \end{pmatrix} = \begin{pmatrix} \text{aileron deflection} \\ \text{rudder deflection} \end{pmatrix} \quad (2.3)$

Similarly for linearized longitudinal motion:

$$\underline{x}_p = \begin{pmatrix} q \\ v \\ \alpha \\ \theta \end{pmatrix} = \begin{pmatrix} \text{pitch rate} \\ \text{velocity} \\ \text{angle of attack} \\ \text{pitch angle} \end{pmatrix} \quad (2.4)$$

and $\underline{u}_p = \begin{pmatrix} \delta_e \\ \delta_T \end{pmatrix} = \begin{pmatrix} \text{elevator deflection} \\ \text{Thrust} \end{pmatrix} \quad (2.5)$

The elements of F and G known to vary with mach number and altitude were provided for a typical fighter aircraft for several flight conditions (FC's). These are provided in Appendix A.

The objective of the research was to find implementable digital algorithms for computing the control signals δ_a , δ_r , δ_e , and δ_T so as to insure uniform and desirable handling quantities for an aircraft flying within the given flight envelope. This was to be performed assuming that during flight, the elements of F and G were not readily available (e.g., as scheduled functions of mach number and altitude).

Because of the need to implement the control system in a digital flight computer, eq. 2.1 was transformed into the equivalent discrete form:

$$\underline{x}_p(k+1) = A_p \underline{x}_p(k) + B_p \underline{u}_p(k) \quad (2.6)$$

where $A_p = e^{F_p T}$

$$B_p = \left(\int_0^T e^{F_p \tau} d\tau \right) G_p$$

T = Sampling period

and $\underline{x}_p(k)$, $\underline{u}_p(k)$ denote \underline{x}_p , \underline{u}_p at time $k \cdot T$
 Eq. 2.6 is a valid representation assuming that F and G do not vary for
 $kT \leq t \leq (k+1)T$ and that the control signal is constant between sample
 times, i.e.,

$$\underline{u}_p(t) = \underline{u}_p(kT) \text{ for } kT \leq t \leq (k+1)T$$

2.1.1.2 Nonlinear Effects

Since the controllers designed using the linearized equations of motion defined in 2.1.1.1 were to be applicable to the actual nonlinear six degree of freedom equations, it is necessary to recognize that the \underline{x}_p and \underline{u}_p vectors of eq. 2.6 are incremental variables which result from a linearization about the trim conditions. Thus if \underline{x}_p^a and \underline{x}_p^t respectively denote the actual and trim aircraft state vectors, and \underline{u}_p^a and \underline{u}_p^t respectively denote the actual and trim controls, then

$$\underline{x}_p = \underline{x}_p^a - \underline{x}_p^t \quad (2.7a)$$

$$\underline{u}_p = \underline{u}_p^a - \underline{u}_p^t \quad (2.7b)$$

However, in actual flight, these trim states will not be explicitly available for computational purposes. Furthermore, direct computation of the trim states themselves is not permissible because of the lack of knowledge of the stability derivatives and the severe computational requirements needed for solving the resulting nonlinear system of equations.

Thus as an alternative it is suggested that the trim states be computed by averaging or low pass filtering, the actual state vector. This is reasonable in view of the expectation that the pilot will almost always be flying the aircraft close to trim conditions. As an example if x_i denotes a particular state of interest,

$$x_i^t = \alpha x_i^t + (1 - \alpha) x_i^a$$

where $0 < \alpha < 1$

If straight and level flight is in progress, then α should be set close to 1.0 resulting in a filter which will effectively smooth out any disturbance. However, if a maneuver is required as indicated by a significant change in the pilot input, then α should be set closer to zero in order for the filter to be able to track the change in trim. These adjustments in α could be made by the computer program in response to sensed stick motion.

If, however, the trim states computed in this manner, are not accurate, in that they do not satisfy the true nonlinear equations of motion, then a bias term must be added to eq. 2.6 so as to account for the additional errors. Thus controls would have to be computed for the system:

$$\underline{x}_p(k+1) = A_p \underline{x}_p(k) + B_p \underline{u}_p(k) + \underline{C} \quad (2.8)$$

where elements in the \underline{C} vector should be identified along with the elements A_p and B_p .

2.1.1.2 Actuator Dynamics

In order to more effectively evaluate the developed control algorithms it is necessary to include the influence of the actuator dynamics in the equation of motion, 2.1. This effect can be modeled by placing in series between each component of the control vector \underline{u} and the aircraft dynamics, a set of secondary and primary actuator dynamics^p defined by the equations:

$$\ddot{\delta}_{e_{sec}} = -2 \zeta_{sec} \omega_{sec} \dot{\delta}_{e_{sec}} - \omega_{sec}^2 \delta_{e_{sec}} + \omega_{sec}^2 \delta_{e_c} \quad (2.9a)$$

$$\ddot{\delta}_{a_{sec}} = -2 \zeta_{sec} \omega_{sec} \dot{\delta}_{a_{sec}} - \omega_{sec}^2 \delta_{a_{sec}} + \omega_{sec}^2 \delta_{a_c} \quad (2.9b)$$

$$\ddot{\delta}_{r_{sec}} = -2 \zeta_{sec} \omega_{sec} \dot{\delta}_{r_{sec}} - \omega_{sec}^2 \delta_{r_{sec}} + \omega_{sec}^2 \delta_{r_c} \quad (2.9c)$$

for the aircraft considered, $\zeta_{sec} = .5$ and $\omega_{sec} = 100$.

$$\dot{\delta}_{e_c} = (\delta_{e_{sec}} - \delta_{e_c})/\tau_1 \quad (2.10a)$$

$$\dot{\delta}_{a_c} = (\delta_{a_{sec}} - \delta_{a_c})/\tau_2 \quad (2.10b)$$

$$\dot{\delta}_{r_c} = (\delta_{r_{sec}} - \delta_{r_c})/\tau_3 \quad (2.10c)$$

$$\dot{\delta}_{T_c} = (\delta_{T_{sec}} - \delta_{T_c})/\tau_4 \quad (2.10d)$$

For the aircraft of interest, $\tau_1 = 1/12.5$, $\tau_2 = 1/30$, $\tau_3 = 1/25$, and $\tau_4 = 1/5$. The computed control signals $\delta_{e_c}^1$, $\delta_{a_c}^2$, $\delta_{r_c}^3$ are thus in reality applied to the secondary actuator dynamics, eqs. 2.9; the outputs of which, $\delta_{e_{sec}}$, $\delta_{a_{sec}}$, $\delta_{r_{sec}}$ are then in turn applied to the primary actuators, eqs. 2.10, to yield the aircraft input vector \underline{u}_p . Note that the thrust input requires only the primary actuator dynamics defined by eq. 2.10d.

These actuator equations can be used in a test simulation in order to determine the effects of neglecting them in the design of \underline{u}_p , or they can actually be incorporated into eq. 2.1 and used for direct computation of

the actuator inputs ($\delta_{e_{sec}}$, $\delta_{a_{sec}}$, $\delta_{r_{sec}}$, δ_{T_c}).

2.1.3 Sensor and Bending Modes

In designing the adaptive control system, it was necessary to take into account the effects of sensor noise and structural bending modes. These were modelled as correlated noise sequences which were added to the state variables in order to form the measurements, y_i . More precisely

$$\underline{y}_p(k) = \underline{x}_p(k) + \underline{n}(k) \quad (2.11)$$

where \underline{x}_p as before denotes the aircraft state vector
 \underline{n} represents the correlated measurement noise sequence
and \underline{y}_p denotes the measurement vector.

Characteristics of the measurement noise are given in Appendix B.

2.1.4 Disturbances

In evaluating the performance of the adaptive controller, it was necessary to take into account the response to wind disturbances and the requirements for a deterministic dither signal to aid identification.

To evaluate the effects of wind, the longitudinal response to vertical disturbances was considered by simulating an angle of attack perturbation equal to the ratio of the wind velocity to the aircraft velocity. The power spectrum of the wind velocity $W(t)$ was defined by:

$$\phi_{WW}(\omega) = \frac{\sigma^2}{\pi} \frac{L}{V_0} \left[\frac{4}{4 + \left(\frac{L}{V_0} \omega\right)^2} \right] \quad (2.12)$$

where $L = 762 \text{ m (2500 ft.)}$
 $\sigma =$ standard deviation of the wind velocity in m/sec
and $V_0 =$ aircraft velocity

Thus the angle of attack perturbation $\Delta\alpha$ can be written as:

$$\Delta\alpha = W/V_0 \quad (2.13)$$

and the modified longitudinal equations of motion become:

$$\frac{d}{dt} \begin{pmatrix} q \\ v \\ \alpha \\ \theta \end{pmatrix} = A_p \begin{pmatrix} q \\ v \\ \alpha \\ \theta \end{pmatrix} + \begin{pmatrix} A_p(1,3) \\ A_p(2,3) \\ A_p(3,3) \\ 0 \end{pmatrix} \Delta\alpha + B_p \begin{pmatrix} \delta_e \\ \delta_T \end{pmatrix} \quad (2.14)$$

With regard to computed or deterministic dither signals for aiding identification, it was necessary to take into account that the resulting behavior be imperceptible to the pilot. This fact was modelled by constraining maximum lateral acceleration to be less than 0.03 g and maximum longitudinal acceleration to be less than 0.1g in response to commanded dither inputs.

2.1.5 Desired Behavior

Inherent to the effectiveness of any adaptive control system is the capability for rapidly assessing the performance and making the necessary modifications to the control gains. One such procedure that fits these requirements and at the same time has the potential for insuring uniform handling qualities is the concept of model following control. This concept has been of interest to many investigators over the past few years.^{8,9,10,11,12} In fact, relative to these efforts, Erzberger¹³ has published a set of conditions under which the output of the process can be made identical to the output of the model.

Being that the ideal objective of model following flight control is to force the aircraft to respond as the model would to a given pilot command, it is often desirable to simulate the online model dynamics in the flight computer and to generate the aircraft control signal using the actual aircraft states, the pilot input commands, and the model states. This situation is sometimes referred to as the pilot's flying of a computer with the computer flying the aircraft.

More precisely the model following problem can be stated as follows:

Given the aircraft dynamics:

$$\underline{x}_p(k+1) = A_p \underline{x}_p(k) + B_p \underline{u}_p(k) \quad (2.15)$$

where $\underline{x}_p(k)$ is the aircraft (n x 1) state vector at sample time k

$\underline{u}_p(k)$ is the (m x 1) control vector

and A_p, B_p are matrices with the appropriate dimensions;

find the control $\underline{u}_p(k)$ such that the process state vector $\underline{x}_p(k)$ approximates "reasonably well" some model's state vector $\underline{x}_m(k)$ defined by the equation:

$$\underline{x}_m(k+1) = A_m \underline{x}_m(k) + B_m \underline{u}_m(k) \quad (2.16)$$

where $\underline{x}_m(k)$ is the (n x 1) model state vector

$\underline{u}_m(k)$ is the (m x 1) pilot input

and A_m, B_m are matrices of the appropriate dimensions.

The desired behavior for the aircraft considered in this study was defined by the following continuous models specified by NASA.

LATERAL MOTION

$$\frac{d}{dt} \begin{pmatrix} p \\ r \\ \beta \\ \phi \end{pmatrix} = \begin{pmatrix} -10. & 0 & -10 & 0 \\ 0 & -0.7 & 9 & 0 \\ 0 & -1. & -0.7 & 0 \\ 1. & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p \\ r \\ \beta \\ \phi \end{pmatrix} + \begin{pmatrix} 20 & 2.8 \\ 0 & -3.13 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \delta_{a_m} \\ \delta_{r_m} \end{pmatrix} \quad (2.17)$$

eigenvalues = (0, -10., -0.7 ± j3.)

LONGITUDINAL MOTION

$$\frac{d}{dt} \begin{pmatrix} q \\ v \\ \alpha \\ \theta \end{pmatrix} = \begin{pmatrix} -1.70 & 0 & -9.87 & 0 \\ 0 & -0.1 & 0 & 0 \\ 1 & 0 & -0.5 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} q \\ v \\ \alpha \\ \theta \end{pmatrix} + \begin{pmatrix} -9.87 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \delta_{e_m} \\ \delta_{T_m} \end{pmatrix} \quad (2.18)$$

eigenvalues = (0., -0.1, -1.1 ± j3.1)

As shown in fig. 2.1a an aileron step command, while not affecting sideslip and yaw rate, results in an overdamped roll rate response with a settling time of about 0.5s. However, a rudder step does affect all states as shown in fig. 2.1b. With regard to the desired longitudinal response, figs. 2.2a and 2.2b show that while a thrust command affects only the

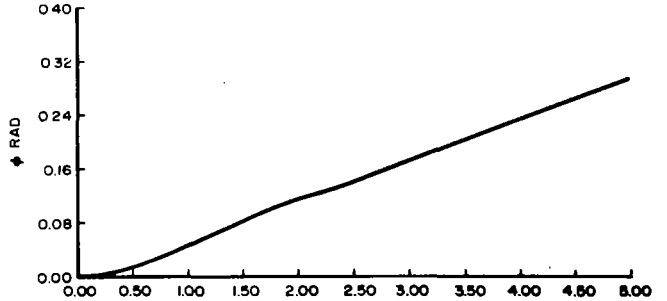
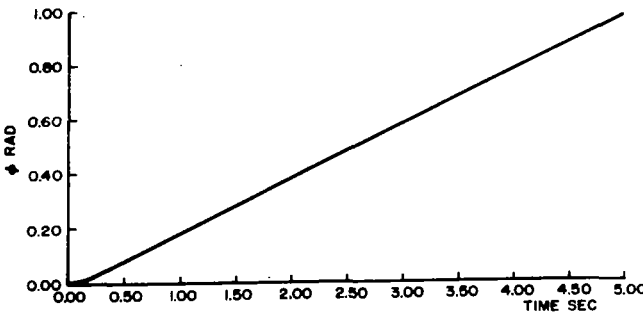
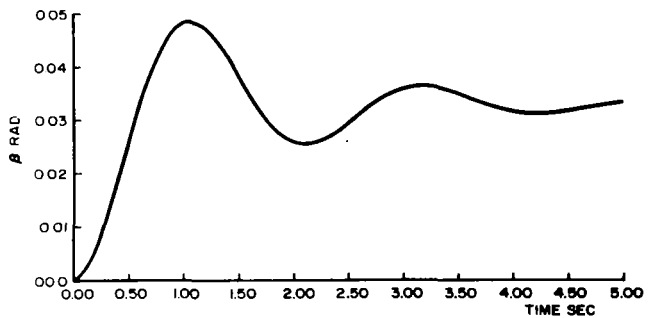
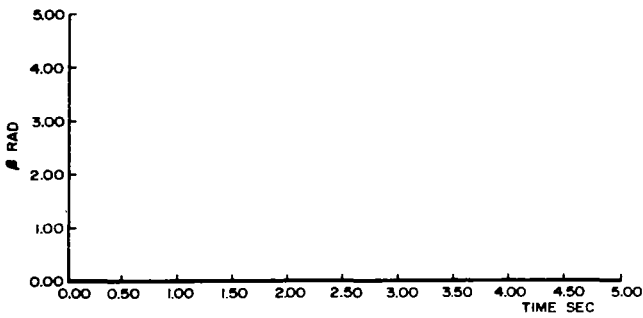
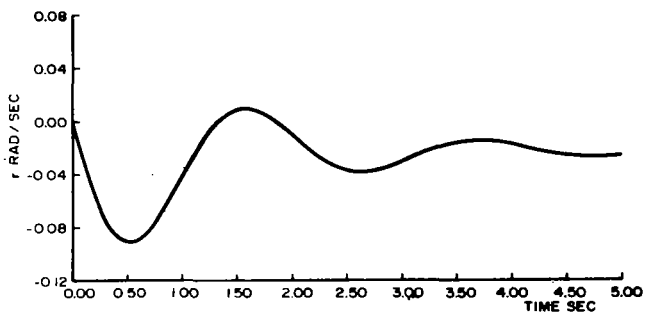
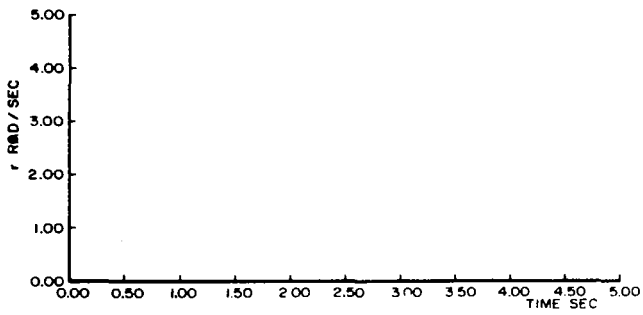
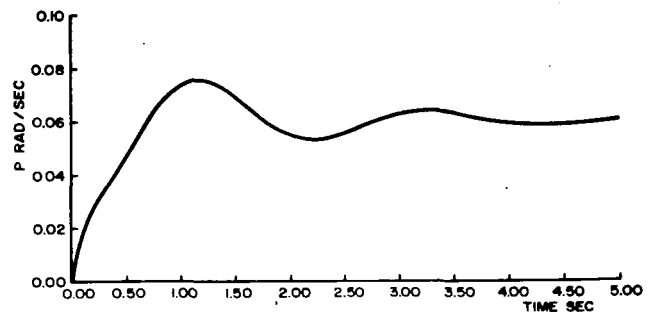
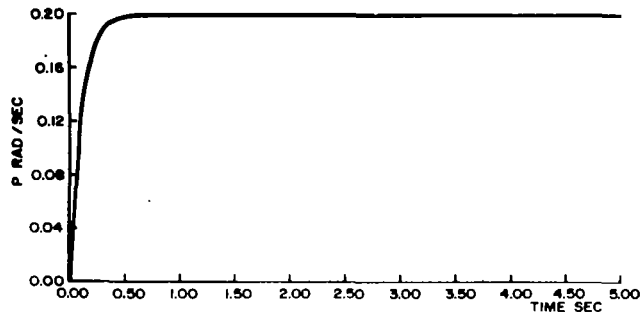


FIG. 2.1a $\theta_0 = 0.1$ RAD
 $\theta_r = 0$

FIG. 2.1b $\theta_0 = 0$
 $\theta_r = 1$ RAD

FIG. 2.1 LATERAL MODEL RESPONSES

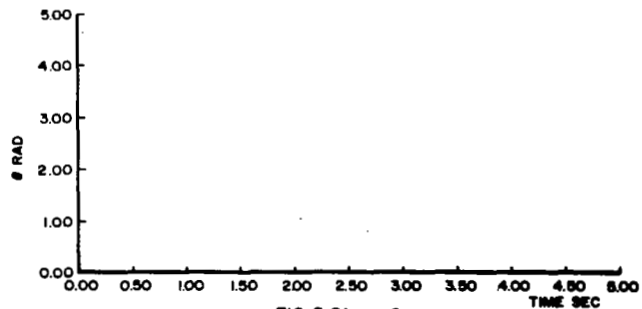
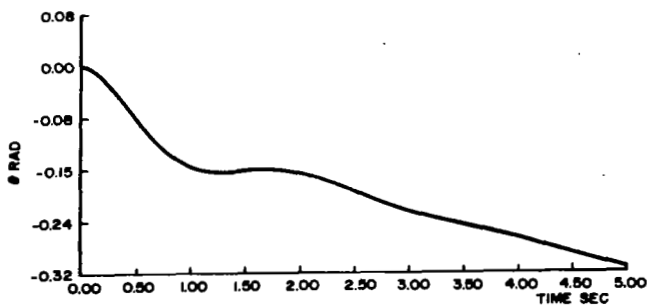
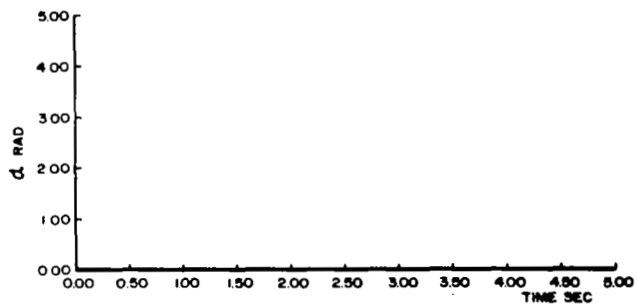
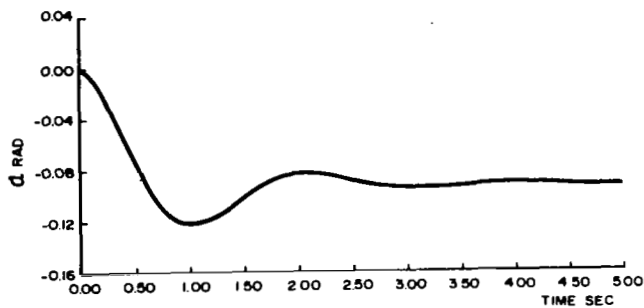
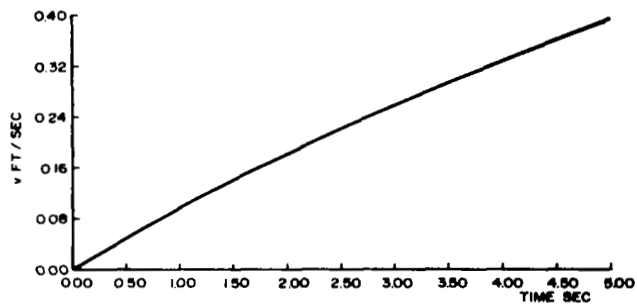
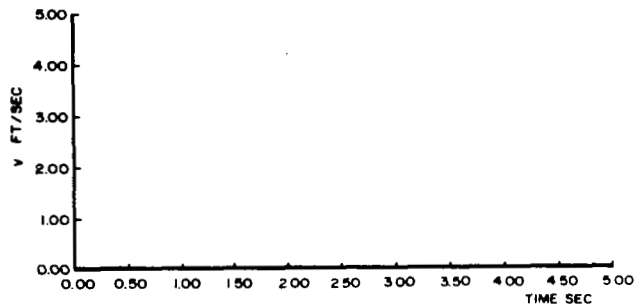
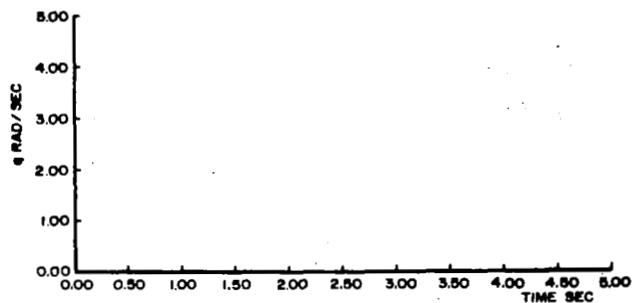
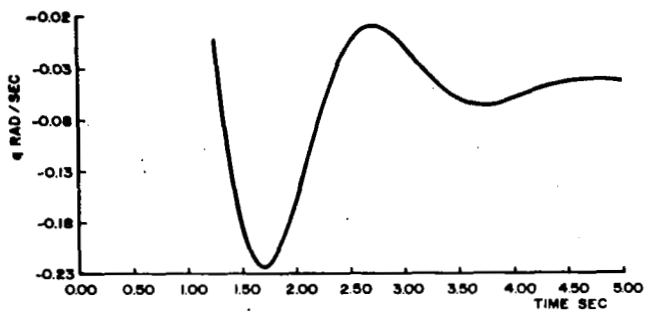


FIG. 2.2a $\theta_0 = 1$ RAD
 $\delta_T = 0$

FIG. 2.2b $\theta_0 = 0$
 $\delta_T = 0.1\%$

FIGURE 2.2 LONGITUDINAL MODEL RESPONSES

velocity, an elevator command will affect all states except for the velocity.

2.2 Adaptive Control System Representation

When designing an algorithm to be implemented, practical considerations must have influence on the trade-off between accuracy and simplicity. In a digital adaptive flight controller, one of the prime practical restrictions is the size and speed of the online digital computer. This will affect both the timing and storage requirements of the adaptive system shown in block diagram form in fig. 2.3. The smallest time interval is the control computation interval and typically is between 0.03 and 0.20 seconds. State estimation if used must be as fast, since the state is used by the feedback controller in calculating the new control.

The two larger intervals involve gain adaptation. The parameter identification interval must not be longer than the gain update interval, since the gain update algorithm requires the new parameter estimates. If parameter estimation alone is performed, then it is possible to have the identification interval greater than the control period; however, if states and parameters are to be estimated simultaneously, then it is necessary that these be equal.

The various functions to be implemented are discussed below.

2.2.1 Controller Formulation

In designing an adaptive control system, it is necessary to first give consideration to the design of either an explicit or an implicit adaptation algorithm; the differences being that:

- . In explicit adaptation, online estimates of the aircraft parameters are used for gain adjustment.
- . In implicit adaptation, some measure of the error between the actual and the desired state trajectories is used for gain adjustment. No explicit parameter identification is used.

Previous studies have indicated the advantages of explicit adaptation if gain magnitudes are constrained and if large parameter variations are to be expected.⁷ Furthermore no implicit adaptive control system which has been developed to date can guarantee stability unless Erzberger's¹³ conditions for perfect model following hold.^{8,9} In view of these items, this study has concentrated on the analysis, synthesis, and subsequent evaluation of explicit adaptive controllers for inflight implementation. However, in parallel with these efforts, some consideration has been given to developing implicit adaptive controller which do guarantee stability at least in the sense of a bounded error between the plant and model states. These results are discussed in Section 3.1.2.

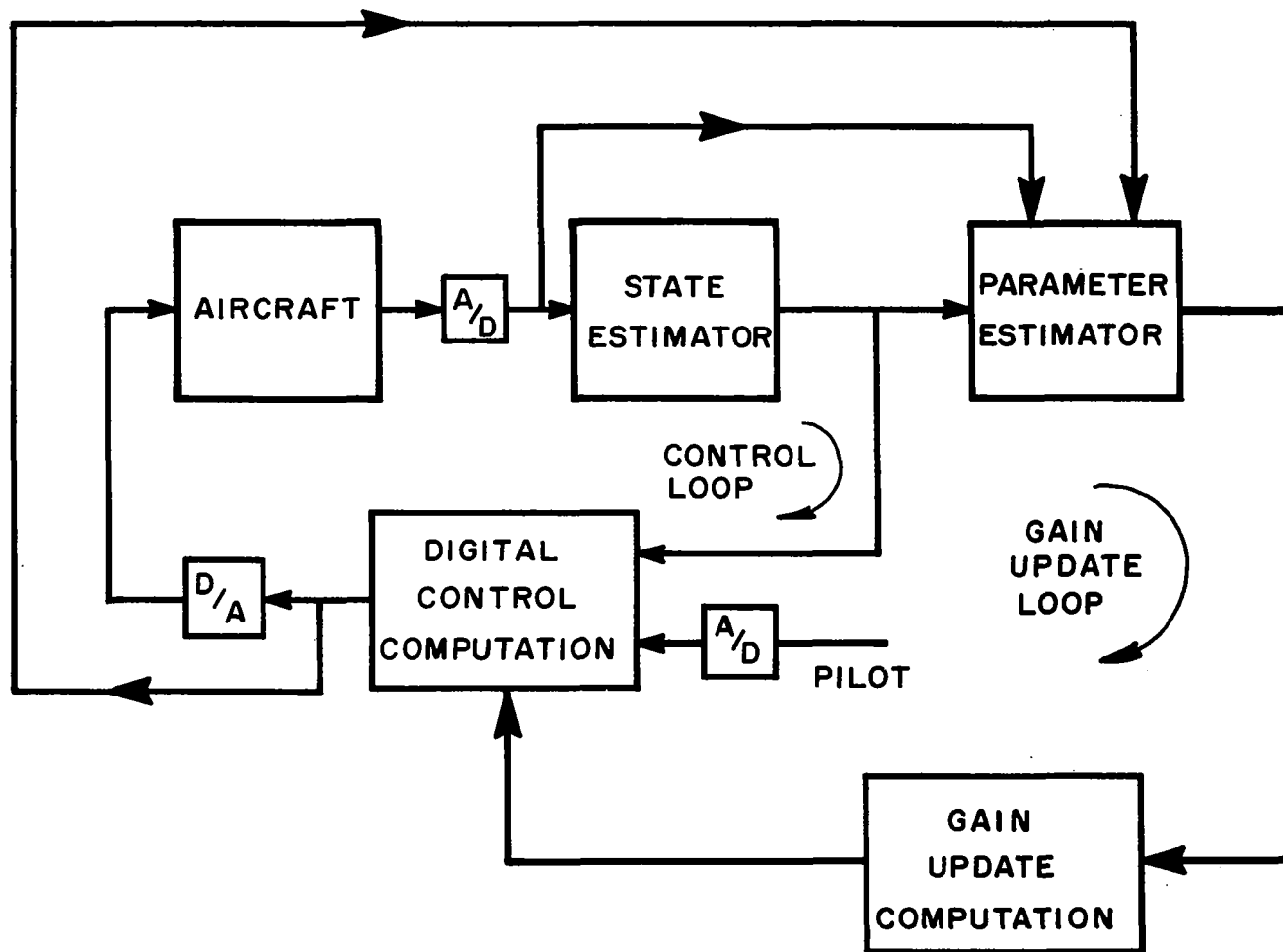


FIG. 2.3 ADAPTIVE CONTROLLER STRUCTURE

In designing a model reference adaptive control system, it is necessary to develop a control algorithm that is meaningful in terms of performance and is readily adjustable online in response to parameter variations. To this effect quadratic performance indices which simultaneously weight the error,

$$\underline{e} = \underline{x}_m - \underline{x}_p \quad (2.19)$$

and the control vector \underline{u}_p were minimized subject to the satisfaction of the state equations.

If the model state vector is defined as in eq. 2.15, then the error vector can be written as:

$$\begin{aligned} \underline{e}(k+1) = \underline{x}_m(k+1) - \underline{x}_p(k+1) &= (A_m \underline{x}_m(k) + B_m \underline{u}_m(k)) \\ &\quad - (A_p \underline{x}_p(k) + B_p \underline{u}_p(k)) \end{aligned} \quad (2.20)$$

This representation of the model dynamics as part of the state equation leads to a real model following control configuration which requires the model state vector \underline{x}_m for implementation.⁶

Alternatively the model dynamics can be incorporated directly into the performance index by defining

$$\underline{e}(k+1) = (A_m \underline{x}_p(k) + B_m \underline{u}_m(k)) - (A_p \underline{x}_p(k) + B_p \underline{u}_p(k)) \quad (2.21)$$

This definition which corresponds to re-initializing the model state at each step to the aircraft state results in an implicit model following controller which is independent of the model state \underline{x}_m .

Although real model following is more complex in that it is necessary to initialize the model states equal to those of the aircraft, it was anticipated and shown by experiment that it is more effective in compensating for unknown parameters and disturbances. For actual implementation of such a system, it is suggested that the model state vector be reset equal to the aircraft state vector whenever a significant change in pilot command \underline{u}_m is detected. Alternately, this initialization procedure might be performed sequentially with a period equal to twice the largest closed loop time constant.

2.2.2 Gain Adaptation

Since the controller parameters must be readily adjustable online in response to identified parameters variations, it would be ideal if the control gains were easily computable algebraic functions of the parameters in the aircraft equations of motion.

This type of controller will in fact result if a single stage performance index of the form:

$$J(k) = \underline{e}^T(k) Q \underline{e}(k) + \underline{u}_p^T(k) R \underline{u}_p(k); Q, R \geq 0 \quad (2.22)$$

is used.^{6,14,15} However, since this index results in a set of control gains which do not guarantee stability, it may be desirable to incorporate Chan's modification¹² and include an error feedback term of the form $K_e \underline{e}$, where K must be determined so as to stabilize the closed loop aircraft matrix $(A_p - B_p K)$.¹⁶ With this modification, the feedforward gains from the model can still be computed online as algebraic functions of A_p and B_p , but the gain K must either be computed in an iterative manner so as to stabilize (A_p, B_p) or must be determined a priori offline so as to (if possible) stabilize (A_p, B_p) over the entire flight envelope.

An alternate procedure for computing the control gain is to use an infinite time quadratic performance index of the form

$$J = \sum_{k=0}^{\infty} [\underline{e}^T(k) Q \underline{e}(k) + \underline{u}_p^T(k) R \underline{u}_p(k)] \quad (2.23)$$

when $Q \geq 0, R \geq 0$.

This approach is attractive in that it yields a constant set of feedback gains that stabilize the closed loop control system.^{6,17} However, since the gains require solution of a nonlinear matrix Riccati equation, an online iterative procedure must be used for adaptation purposes.¹⁷

2.2.3 Estimation

Because of the need for using both the state vector \underline{x}_p and the aircraft matrices A_p, B_p for explicit adaptive control computation, it was necessary to include estimation logic into the system as shown in fig. 2.3.

In designing estimation algorithms, attention must be given to the measurement noise characteristics. Whereas a relatively large variance necessitates the use of an identifier with a long memory to achieve smoothing, a small variance will enable the use of a short memory identifier that will be more responsive to parameter variations.

Furthermore in a digital environment, it is advantageous to identify the unknown parameters of the discrete transition matrix itself rather than the physical stability derivatives. This follows because the discrete transition matrix is a highly nonlinear function of the stability derivatives making identification rather difficult. Since the discrete control law itself is directly related to the discrete transition matrix, the latter would ultimately have to be recomputed using the stability derivative estimates.

Finally, of importance is the performance of the identification procedures under closed loop control. Because such control often results in transient behavior for only a very small amount of time and steady state behavior for a relatively large amount of time, there may not be sufficient excitation to allow accurate enough identification. Thus the need for an induced dither signal must be examined. Such dither could in fact be produced

by feeding back for control computation the noisy state measurements themselves rather than filtered state estimates. In any event the performance of the identifier should ultimately be measured by the overall behavior of the adaptive control system rather than the individual estimates it produces for the various parameters. This follows from the fact that not all states are equally excited by any given pilot input command. Hence various motions will be completely decoupled or very insensitive to the specific values obtained for some of the parameters. Thus while there may not be sufficient excitation present to accurately track a parameter, its value may not be influential to the maneuver being undertaken. Accurate tracking of all parameters continuously will only be possible if dither can be acceptably introduced into the motion of the aircraft.

An additional factor which can aid the identification is the effect of having an improperly identified aircraft resulting in the application of erroneous control gains which in turn can cause erratic motion. However, since identification works best in the presence of such motion, it is anticipated that the adaptive system will rapidly correct for large inaccuracies.

3. PROBLEM SYNTHESIS

3.1 Control Computation Procedures

3.1.1 Explicit Adaptive Controllers

Explicit adaptive control logic directly utilizes online parameter estimates for adjusting the control gains; thus, relative to implicit adaptation, better stability margins with lower gain requirements were anticipated. Two explicit adaptive control systems were subsequently designed and tested.

3.1.1.1 Adaptive Optimal Linear Regulator Logic

Because ease of implementation was an important consideration, a linear feedback structure with constant gains (for a given flight condition) was very desirable. To design such a system, infinite time quadratic performance indices were minimized for the system defined by eqs. 2.15, 2.16. Such an index generally consists of some positive semi-definite quadratic function of the model following error $(\underline{x}(k) - \underline{\hat{x}}(k))$ balanced against a positive quadratic function of the control $\underline{u}_p(k)$.

Alternatively it was shown by Asseo¹⁰ that penalizing the control rate $\dot{\underline{u}}_p(t)$ rather than the control itself yields reduced sensitivity to plant parameter variations. For the continuous case this necessitates treating the plant control $\underline{u}_p(t)$ as an additional state variable with corresponding equation,

$$\dot{\underline{u}}_p(t) = \underline{v}_p(t) \quad (3.1)$$

thus resulting in a type one controller.

Relative to a type zero controller, a type one controller yields improved steady state performance, reduced sensitivity, and the capability for penalizing the control rate itself. Thus the discrete version of the type one controller was defined by replacing the integrator by an accumulator (i.e., a unit delay with unity feedback). The corresponding state equation then becomes:

$$\underline{u}_p(k+1) = \underline{u}_p(k) + \underline{v}_p(k) \quad (3.2)$$

The optimization problem used for defining the controller was:

Minimize:

$$J = \frac{1}{2} \sum_{k=0}^{\infty} \{ (\underline{x}_p(k) - \underline{x}_m(k))^T Q (\underline{x}_p(k) - \underline{x}_m(k)) + \underline{v}_p^T(k) R \underline{v}_p(k) + \underline{u}_p^T(k) S \underline{u}_p(k) \} \quad (3.3a)$$

subject to:

$$\underline{x}_p(k+1) = A_p \underline{x}_p(k) + B_p \underline{u}_p(k) \quad (3.3b)$$

$$\underline{u}_p(k+1) = \underline{u}_p(k) + \underline{v}_p(k) \quad (3.3c)$$

$$\underline{x}_m(k+1) = A_m \underline{x}_m(k) + B_m \underline{u}_m(k) \quad (3.3d)$$

$$\underline{u}_m(k+1) = \underline{u}_m(k) \quad (3.3e)$$

This cost index (3.3a) represents the simultaneous penalization of model following error, control rate, and the control vector itself, each weighted according to the designer's choice of Q, R, and S. Note that an additional state equation (3.3e) is used to represent the pilot input $\underline{u}_m(k)$ as a step function. This results in a set of control gains independent of the step magnitude which in turn can be accounted for by specifying it as an initial condition, $\underline{u}_m(0)$.

With regard to model initialization, since it is desirable upon application of a pilot input \underline{u}_m that the plant and model be at the same state, $\underline{x}_m(0)$ was set equal to $\underline{x}_p(0)$. Furthermore, to reflect the fact that control surfaces cannot change instantaneously, $\underline{u}_p(0)$ was selected as zero. Finally, it was determined through simulation that weighting \underline{u}_p in eq. (3.3a) was not necessary in view of the weighting on \underline{v}_p . Thus S^p has been replaced by zero for the remainder of the development.

This optimization problem can now be cast into the form:

Minimize:

$$J = \frac{1}{2} \sum_{k=0}^{\infty} [\underline{x}^T(k) Q' \underline{x}(k) + \underline{u}^T(k) R' \underline{u}(k)] \quad (3.4a)$$

subject to

$$\underline{x}(k+1) = A \underline{x}(k) + B \underline{u}(k) \quad (3.4b)$$

where:

$$\underline{x}(k) = \begin{bmatrix} \underline{x}_p(k) \\ \underline{u}_p(k) \\ \underline{x}_m(k) \\ \underline{u}_m(k) \end{bmatrix} \quad \underline{u}(k) = \underline{v}_p(k) \quad A = \begin{bmatrix} A_p & B_p & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & A_m & B_m \\ 0 & 0 & 0 & I \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ I \\ 0 \\ 0 \end{bmatrix}$$

$$Q' = \begin{bmatrix} Q & 0 & -Q & 0 \\ 0 & 0 & 0 & 0 \\ -Q & 0 & Q & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R' = R$$

The existence of a set of optimal control gains for this problem can be demonstrated even though the model, which has been incorporated into the state equations, is not controllable.¹⁹ The control $\underline{u}(k) = \underline{v}_p(k)$ will be a linear feedback law of the form:

$$\underline{u}(k) = -K \underline{x}(k) \quad (3.5)$$

The control gain K is then in turn defined by the relation:

$$K = (B^T P B + R)^{-1} B^T P A \quad (3.6a)$$

where P is specified by the steady state Riccati equation:

$$P = Q + A^T P A - A^T P B (R + B^T P B)^{-1} B^T P A \quad (3.6b)$$

By partitioning eq. 3.5 the control \underline{u}_p can be expressed as:

$$\underline{u}_p(k+1) = \underline{u}_p(k) + \underline{v}_p(k) \quad (3.7)$$

or

$$\underline{u}_p(k+1) = -K_{x_p} \underline{x}_p(k) - K_{x_m} \underline{x}_m(k) - K_{u_m} \underline{u}_m(k) + (I - K_{u_p}) \underline{u}_p(k) \quad (3.8)$$

Similar partitioning of P into:

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{12}^T & P_{22} & P_{23} & P_{24} \\ P_{13}^T & P_{23}^T & P_{33} & P_{34} \\ P_{14}^T & P_{24}^T & P_{34}^T & P_{44} \end{bmatrix}$$

results in the following alternative form of eq. 3.6b

$$P_{11} = Q + A_p^T (P_{11} - P_{12}(P_{22} + R)^{-1} P_{12}^T) A_p \quad (3.9a)$$

$$P_{12} = A_p^T [(P_{11} - P_{12}(P_{22} + R)^{-1} P_{12}^T) B_p + P_{12}(I - (P_{22} + R)^{-1} P_{22})] \quad (3.9b)$$

$$P_{22} = B_p^T [(P_{11} - P_{12}(P_{22} + R)^{-1} P_{12}^T) B_p + P_{12}(I - (P_{22} + R)^{-1} P_{22})] \\ + [(I - (P_{22} + R)^{-1} P_{22}) P_{12}^T] B_p + P_{22}(I - (P_{22} + R)^{-1} P_{22}) \quad (3.9c)$$

$$P_{13} = -Q + A_p^T (P_{13} - P_{12}(P_{22} + R)^{-1} P_{23}) A_m \quad (3.9d)$$

$$P_{14} = A_p^T [P_{14} + (P_{13} - P_{12}(P_{22} + R)^{-1} P_{23}) B_m - P_{12}(P_{22} + R)^{-1} P_{24}] \quad (3.9e)$$

$$P_{24} = B_p^T [P_{14} + (P_{13} - P_{12}(P_{22} + R)^{-1} P_{23}) B_m - P_{12}(P_{22} + R)^{-1} P_{24}] \\ + (P_{23} - P_{22}(P_{22} + R)^{-1} P_{23}) B_m + P_{24} - P_{22}(P_{22} + R)^{-1} P_{24} \quad (3.9f)$$

$$P_{23} = B_p^T (P_{13} - P_{12}(P_{22} + R)^{-1} P_{23}) A_m + (P_{23} - P_{22}(P_{22} + R)^{-1} P_{23}) A_m \quad (3.9g)$$

P_{33} , P_{34} , and P_{44} were not needed for defining the control gains and therefore were excluded from computation.

The specific gains defined by eq. 3.8 may now be expressed as:

$$K_{x_p} = -(P_{22} + R)^{-1} (P_{12}^T A_p) \quad (3.10a)$$

$$K_{u_p} = -(P_{22} + R)^{-1} (P_{12}^T B_p + P_{22}) \quad (3.10b)$$

$$K_{x_m} = -(P_{22} + R)^{-1} P_{23} A_m \quad (3.10c)$$

$$K_{u_m} = -(P_{22} + R)^{-1} (P_{23} B_m + P_{24}) \quad (3.10d)$$

Although the above expressions do appear quite formidable for online evaluation, iterative procedures do exist for solving the Riccati sub-equations (3.9 a, b, c, d, e, f, g). Once the P submatrices are available, evaluation of the gains (eqs. 3.10) is straight forward formula evaluation. The required inverse, which is common to all four gain matrices and thus need only be computed once, is of the same dimension as the control signal. For the aircraft problem being treated, this inversion is of second order, and thus can be performed by simple formula evaluation.

The adaptation logic for this type controller is based upon an online iteration of the partitioned Riccati equation, eqs. 3.9. Since the aircraft parameters vary continuously and relatively little within the anticipated gain update cycles, it can be expected that the exact solution to the corresponding steady state Riccati equation will not vary significantly between gain updates. Thus, if at each gain update time the Riccati equation is initialized with the most recent solution, it is hypothesized that it will be necessary to iterate only a few times to find the proper solution.

Three iterative procedures, as described below, were considered for updating the solution to eq. 3.9.

- Backwards iteration of the time varying Riccati solution

This procedure, which is the simplest to implement, is equivalent to solving backwards in time the Riccati equation corresponding to the finite time linear regulator problem.^{19,20} The P matrix is initialized to any positive semi-definite value at the zeroth iteration and updated at the k^{th} iteration using:

$$P(k+1) = Q + A^T P(k) A - A^T P(k) B (R + B^T P(k) B)^{-1} B^T P(k) A \quad (3.11)$$

To illustrate the role of eq. 3.11 in an adaptive mode, assume that at some time k , $P(k)$ is already available corresponding to the estimates $\hat{A}(k)$ and $\hat{B}(k)$ for A and B respectively. Since A and B will be changing between the k^{th} and $(k+1)^{\text{st}}$ samples, the estimates $\hat{A}(k+1)$, $\hat{B}(k+1)$ will be different from the corresponding values at time k . However, if these differences are not too severe, then it is anticipated that the appropriate value of P which corresponds to the true values of A and B at time k can be approximated by evaluating the right hand side of eq. 3.11 with $P(k)$, $\hat{A}(k+1)$, $\hat{B}(k+1)$.

- Quasilinearization procedure

A quasilinearization procedure proposed by Hewer²¹ for finding the steady state solution of (3.9) can be summarized as follows:

$$P = \lim_{k \rightarrow \infty} V_k$$

where V_k satisfies the linear equation

$$V_k = \phi_k^T V_k \phi_k + L_k^T R L_k + Q \quad (3.12a)$$

and

$$L_k = (R + B^T V_{k-1} B)^{-1} B^T V_{k-1} A \quad (3.12b)$$

$$\phi_k = A - B L_k \quad (3.12c)$$

This procedure requires the selection of L_0 such that $(A - B L_0) = Q_0$ is stable. A means for such an initialization has in fact been given by Kleinman.²² Again this procedure can be applied to adaptation by evaluating the right hand sides of (3.12) with the most recently computed value for V_k and the currently identified values of A and B .

● First Order parameter expansion

If between gain update times, the parameter changes ΔA and ΔB are not too large, then the Riccati matrix corresponding to $A + \Delta A$ and $B + \Delta B$ (i.e., $P(A + \Delta A, B + \Delta B)$) can be expressed as a first order expansion about $P(A, B)$. This will result in a system of $\frac{n(n+1)}{2}$ linear equations which can then be solved for the $n(n+1)/2$ elements of $\Delta P = P(A + \Delta A, B + \Delta B) - P(A, B)$.

These three procedures were evaluated (assuming perfect identification) by:

- . Initializing the P matrix to a value corresponding to a given flight condition and evaluating the convergence to neighboring flight conditions. Typical per sample changes in A and B were considered.
- . Initializing the P matrix to zero and investigating the convergence of the gains as well as the Riccati matrix for various flight conditions. This test is an indication of the ability for adapting to large parameter changes. Because ΔA and ΔB are not used, only the first two procedures were evaluated in this manner.

It was observed during the first evaluation procedure that adaptation based upon the third method, (i.e., first order expansions) was extremely inaccurate relative to the other two procedures, both of which performed equally well in terms of the number of iterations needed for computing the gains to within three figure accuracy. For typical per sample parameter transitions, only one, or at most two, iterations of the first two procedures were needed to insure three or four figure accuracy in the gains. However, because the first procedure required only a formula evaluation, while the second procedure required the solution of the $\frac{n(n+1)}{2}$ components of eq. 3.12a, the first or backwards iteration procedure required less computer time per iteration. Therefore, the backwards iteration procedure was

selected for application to the adaptive system design.

3.1.1.2 Single Stage Adaptive Controller

Development of an implementable digital adaptive control system requires that consideration be given to designing a control algorithm that performs well, and at the same time is easily adjustable online in response to parameter changes. For example, while feedback gains determined from the solution to the linear quadratic optimal control problem can be easily designed offline, adaptation of these gains, as shown in the previous section, requires the online solution of a nonlinear matrix (Riccati) equation.

An alternative to such a design may be feasible if Erzberger's¹³ conditions for perfect model following apply, i.e.:

$$(I - B_p B_p^+) (A_m - A_p) = 0 \quad (3.13a)$$

$$(I - B_p B_p^+) B_m = 0 \quad (3.13b)$$

where B_p^+ is the pseudo-inverse of B_p . If these conditions are satisfied, then the implicit model following controller:

$$\underline{u}_p = B_p^+ (A_m - A_p) \underline{x}_p + B_p^+ B_m \underline{u}_m \quad (3.14)$$

will result in perfect model following.

However, because these conditions of perfect model following are not always attainable in practice and because the gains for perfect model following can be too high, single stage performance indices, which penalize both the model following effort and the control effort, were considered.^{6,23} In particular a performance index of the form

$$J = \underline{e}^T(k+1) Q \underline{e}(k+1) + \underline{u}_p^T(k) R \underline{u}_p(k) \quad (3.15)$$

where

$$\begin{aligned} \underline{e}(k+1) = & (A_m \underline{x}_m(k) + B_m \underline{u}_m(k)) \\ & - (A_p \underline{x}_p(k) + B_p \underline{u}_p(k)) \end{aligned} \quad (3.16)$$

was minimized to yield the real model following control law:

$$\begin{aligned} \underline{u}_p(k) = & [R + B_p^T Q B_p]^{-1} B_p^T Q [A_m \underline{x}_m(k) \\ & - A_p \underline{x}_p(k) + B_m \underline{u}_m(k)] \end{aligned} \quad (3.17)$$

Although this controller is attractive because the control gains can be readily computed online by formula evaluation, the penalization of the behavior only one step into the future makes it impossible to guarantee stability. Thus a modification as suggested by Chan¹² was incorporated. This results in a real model following control law which yields perfect model following if Erzberger's conditions are satisfied and an error that is bounded, otherwise. This controller (for $R=0$ and $Q=I$) has the form:

$$\underline{u}_p = \underline{u}_1 + \underline{u}_2 \quad (3.18)$$

where

$$\underline{u}_1 = +K \underline{e} \quad (3.19a)$$

$$\underline{u}_2 = B_p^+ (A_m - A_p) \underline{x}_m + B_p^+ B_m \underline{u}_m \quad (3.19b)$$

$$\underline{e} = \underline{x}_m - \underline{x}_p$$

In a manner similar to Chan's development for continuous systems, it was shown in ref. 16 that if K is chosen to stabilize $(A_p - B_p K)$, then the above controller will yield stability at least in the sense of boundedness even if the conditions for perfect model following are not satisfied. Combining eqs. 3.18 and 3.19 yields the composite controller:

$$\underline{u}_p = -K \underline{x}_p + K_{x_m} \underline{x}_m + K_{u_m} \underline{u}_m \quad (3.20a)$$

where

$$K_{x_m} = B_p^+ (A_m - A_p) + K \quad (3.20b)$$

and

$$K_{u_m} = B_p^+ B_m \quad (3.20c)$$

Clearly online adjustment of the gains in \underline{u}_2 (eq. 3.19b) can be readily accomplished by simple formula evaluation as updated parameter estimates are received. However, online evaluation of K so as to stabilize $G = (A_p - B_p K)$ may not be as straightforward.

One procedure for stabilizing G is to solve the linear optimal regulator problem:

$$\text{Minimize: } J = \frac{1}{2} \sum_{k=0}^{\infty} \underline{e}^T(k) Q \underline{e}(k) + \underline{u}_1^T(k) R \underline{u}_1(k) \quad (3.21)$$

Subject to:

$$\underline{e}(k+1) = A_p \underline{e}(k) + B_p \underline{u}_1(k) \quad (3.22)$$

Then under the conditions that (A_p, B_p) is a controllable pair, R is positive definite, $Q = DD^T$, and (A_p, D) is observable, it can be shown²⁰ that

$$\underline{u}_1 = K \underline{e} \quad (3.23)$$

where

$$K = + (R + B_p^T P B_p)^{-1} B_p^T P A_p \quad (3.24)$$

Thus for online adaptation purposes, K might be adjusted using one of the procedures discussed in Section 3.1.1.1.

Alternately, it may be possible through a judicious selection of the nominal values for A_p and B_p to find a gain K that stabilizes $G = (A_p - B_p K)$ over the complete flight envelope.

One approach for determining such a gain is to determine a controller which satisfies the guaranteed cost criterion as stated by Chang and Peng²⁴ for continuous systems. In particular, if for a given set of controllable F_p and G_p matrices (appearing in the continuous eq. 2.1), it is possible to find a matrix $P > 0$ satisfying

$$\frac{1}{2} \underline{x}_p^T Q \underline{x}_p + \frac{1}{2} \underline{u}_p^T R \underline{u}_p + \underline{x}_p^T P [F_p \underline{x}_p + G_p \underline{u}_p] \leq 0 \quad (3.25)$$

where

$$\underline{u}_p = - R^{-1} G_o^T P \underline{x}_p \quad (3.26)$$

then the closed loop transition matrix

$$F_p - G_p R^{-1} G_o^T P$$

will be stable for all F_p and G_p in the given set. Furthermore, the cost function J , defined in eq. 2.21, will be less than $\frac{1}{2} \underline{x}_p^T(0) P \underline{x}_p(0)$, the guaranteed cost.²⁴

Substitution of eq. 3.26 into eq. 3.25 yields

$$\frac{1}{2} \underline{x}_p^T [Q + P G_o R^{-1} G_o^T P - 2 P G_p R^{-1} G_o^T P + P F_p + F_p^T P] \underline{x}_p \leq 0$$

This condition holds if P is such that the matrix

$$P F_p + F_p^T P + P [G_o R^{-1} G_o^T - 2 G_p R^{-1} G_o^T] P + Q$$

is negative semi-definite for all F_p and G_p . Because F_p and G_p appear separately in this expression, a P matrix satisfying (3.25)^p can be determined from the matrix Riccati equation

$$P F_o + F_o^T P - P G_o R^{-1} G_o^T P + Q = 0 \quad (3.27)$$

if

$$F_o = \begin{pmatrix} f_1 & 0 & 0 & 0 \\ 0 & f_2 & 0 & 0 \\ 0 & 0 & f_3 & 0 \\ 0 & 0 & 0 & f_4 \end{pmatrix}$$

where: $f_1 > f_2 > f_3 > f_4 >$ maximum absolute eigenvalue of $\frac{(F_p + F_p^T)}{2}$
and G_o is selected such that

$$G_o F^{-1} G_o^T \leq G_o R^{-1} G_o^T \text{ for all } G_p \quad (3.28)$$

This latter condition is satisfied if G_p can be expressed as the product of a nominal matrix G_n and a positive definite square diagonal matrix G_D , i.e.,

$$G_p = G_n G_D \quad (3.29)$$

Therefore, for the systems defined in eqs. 2.2-2.4, G_D would be of the form

$$G_D = \begin{pmatrix} g_1 & 0 \\ 0 & g_2 \end{pmatrix} \quad 0 < g_1 < g_2 \quad (3.30)$$

and G_o can be chosen as

$$G_o = \begin{pmatrix} g_1^* & 0 \\ 0 & g_1^* \end{pmatrix} \quad (3.31)$$

where g_1^* is the smallest value of g_1 with respect to all permissible variations over the flight envelope. This representation as defined in eq. 3.29 does not appear unreasonable in view of the given values for G_p over the flight envelope of interest.

Application of these procedures to the discrete optimization problem is not as straightforward because it is not possible to separate the effects of selecting A_0 from the effects of selecting B_0 . Thus it is recommended that either:

- . The continuous feedback gains (eq. 3.26) be used directly in the discrete system. This should be stabilizing if the sampling time is not too large.

or

- . That the equivalent discrete system be found corresponding to (F_0, G_0) . Computation of the control gains would then proceed by applying eqs. 3.22-3.24.

It should be noted that in applying these procedures, the resulting feedback gain may not give desirable transient behavior for all flight conditions. This can result from the desire to trade off a constant stabilizing gain with a time varying gain optimized by some online iterative procedure.

3.1.1.3 Comparative Discussion

Relative to the adaptive optimal regulator controller and the single stage adaptive controller respectively discussed in 3.1.1.1 and in 3.1.1.2, the following points should be noted:

- . The gain update logic for the single stage algorithm is more easily implemented. This is especially true if a satisfactory constant feedback gain can be determined that stabilizes the open loop dynamics over a fairly wide portion of the flight envelop. If however this is not possible and an online update of the Riccati equation (3.24) is desired, then it should be noted that for a fourth order system, this would correspond to the need to update only 10 equations rather than the 69 equations (3.9) for the adaptive optimal regulator controller.
- . The optimal regulator is more amenable to a redesign for implicit model following, since the stabilized single stage controller does require feedforward of the model state vector.
- . The optimal regulator design allows penalization of the control rate and results in a type one controller which should yield better performance with respect to steady state model following in the presence of uncertain parameters.

3.1.2 Implicit Adaptive Controllers

Implicit adaptive control algorithms are attractive for implementation because they do not require the explicit use of parameter estimates. Consequently the problem of how to satisfactorily implement online identification is eliminated.

Although several implicit adaptive controller have been proposed for linear model reference systems, no assurance of stability can be cited unless the plant and model satisfy certain structural conditions. Typical examples include the "MIT method", which cannot in general be shown to be stable,²⁵ the procedure of Winsor and Roy which requires the ability to independently adjust each element of the plant matrices,⁸ and Landau's hyperstability approach⁹ which yields asymptotic stability if the plant and model matrices conform according to Erzberger's conditions for perfect model following¹³.

However, since these conditions for perfect model following are not valid for the problem as presented in Section 2.0, it was necessary to determine what, if any, modifications were required in order to guarantee stability, at least in the sense of a bounded error, if an implicit adaptive control algorithm is to be used.

To this effect, since Landau's algorithm was general enough to have been previously applied to an aircraft model reference control system⁹, its use was attempted in a simulation of the problem defined in Section 2.0. Results (presented in Section 4.1.2) indicated that even though the conditions of perfect model following did not apply, the adaptive control law was capable of improving the performance in the presence of unknown parameters and yielding bounded errors.

Consequently an analytical study was performed in order to determine if these results could have been predicted. This was done by defining the controller to be of the form:

$$\begin{aligned} \underline{u}_p(t) = & -K_{x_p} \underline{x}_p + K_{x_m} \underline{x}_m + K_{u_m} \underline{u}_m + \dot{\phi}(t) \underline{x}_p + \dot{\psi}(t) \underline{u}_m \\ & + \phi(t) \underline{x}_p + \psi(t) \underline{u}_m \end{aligned} \quad (3.32)$$

and applying the Lyapunov function:

$$V = \underline{e}^T P \underline{e} + \text{Trace} (\phi - \tilde{A}) Q^{-1} (\phi - \tilde{A})^T + \text{Trace} (\psi - \tilde{B}) R^{-1} (\psi - \tilde{B})^T \quad (3.33)$$

where $\underline{e} = \underline{x}_m - \underline{x}_p$

$$\dot{\phi} = D \underline{e} \underline{x}_p^T Q \quad (3.34a)$$

$$\dot{\psi} = D \underline{e} \underline{u}_m^T R \quad (3.34b)$$

P , Q , R are positive definite symmetric matrices, and \tilde{D} , \tilde{A} , and \tilde{B} are matrices to be selected so as to insure stability.

Taking the time derivative and selecting $D = G_p^T P$, and using eqs. 2.15 and 2.16, yields:

$$\begin{aligned} \dot{V} = & 2 \underline{e}^T [+ P F_p + P G_p K_{x_p} + D^T \tilde{A}] \underline{e} \\ & - 2 \underline{e}^T P G_p D \underline{e} (\underline{x}_p^T Q \underline{x}_p) \\ & - 2 \underline{e}^T P G_p D \underline{e} (\underline{u}_m^T R \underline{u}_m) \\ & + 2 \underline{e}^T P [F_m - F_p - G_p K_{x_p} - G_p K_{x_m} + D^T \tilde{A}] \underline{x}_m \\ & + 2 \underline{e}^T [P G_m - P G_p K_{u_m} - D^T \tilde{B}] \underline{u}_m \end{aligned} \quad (3.35)$$

Note that with $D = G_p^T P$, the second two terms will be negative definite, and the first term will become:

$$2 \underline{e}^T [P F_p + P G_p (K_{x_p} - \tilde{A})] \underline{e}$$

To guarantee that this term is negative definite, it is only necessary to show that there exists an \tilde{A} such that

$$P F_p + P G_p (K_{x_p} - \tilde{A}) < 0 \quad (3.36)$$

since \tilde{A} does not appear in the control law. The existence of such an \tilde{A} is evident since, the above requirement for negative definiteness corresponds to finding the eight elements of K_{x_p} such that the determinants of the four principle minors of

$$- (P F_p + P G_p K_{x_p}) - (P F_p + P G_p K_{x_p})^T$$

are all positive.

Since the last two terms of \dot{V} involve products of the bounded components of the model state and control with components of the error vector, there will exist values of \underline{e} such that the first three terms of \dot{V} will dominate.

Consequently regardless of the initial gain values, K_{x_p} , K_{x_m} , K_{u_m} , the control law defined by eqs. 3.32 and 3.34 will result in stability in the sense of bounded error. However, since $D = G_p^T P$, this controller requires knowledge of G_p which in fact may be unavailable.

This can be avoided by defining the augmented state vectors

$$\underline{x}_p^{ag} = (\underline{x}_p, \underline{u}_p)^T \quad (3.37a)$$

$$\underline{x}_m^{ag} = (\underline{x}_m, \underline{u}_m)^T \quad (3.37b)$$

with state equations:

$$\dot{\underline{x}}_p^{ag} = F_p^a \underline{x}_p^{ag} + G_p^a \dot{\underline{u}}_p \quad (3.38a)$$

$$\dot{\underline{x}}_m^{ag} = F_m^a \underline{x}_m^{ag} + G_m^a \dot{\underline{u}}_m \quad (3.38b)$$

where

$$F_p^a = \begin{pmatrix} F_p & G_p \\ 0 & 0 \end{pmatrix} \quad G_p^a = \begin{pmatrix} 0 \\ \underset{\substack{\longleftrightarrow \\ m}}{I} \end{pmatrix} \begin{matrix} \updownarrow n \\ \updownarrow m \end{matrix} \quad (3.39a)$$

$$F_m^a = \begin{pmatrix} F_m & G_m \\ 0 & 0 \end{pmatrix} \quad G_m^a = \begin{pmatrix} 0 \\ \underset{\substack{\longleftrightarrow \\ m}}{I} \end{pmatrix} \begin{matrix} \updownarrow n \\ \updownarrow m \end{matrix} \quad (3.39b)$$

An analysis similar to that of eq. 3.33-3.35, but now performed on the augmented system results in the stabilizing controller

$$\begin{aligned} \dot{\underline{u}}_p = & -K_{x_p} \underline{x}_p^{ag} + K_{x_m} \underline{x}_m^{ag} + K_{u_m} \dot{\underline{u}}_m \\ & + \dot{\phi}(t) \underline{x}_p^{ag} + \dot{\psi}(t) \underline{u}_m + \phi(t) \underline{x}_p^{ag} + \psi(t) \dot{\underline{u}}_m \end{aligned} \quad (3.40)$$

$$\text{where } D = P G_p^a \quad (3.41a)$$

$$\underline{e}^{ag} = \underline{x}_m^{ag} - \underline{x}_p^{ag} \quad (3.41b)$$

$$\dot{\phi} = D \underline{e}^{ag} \underline{x}_p^{ag^T} Q^{-1} \quad (3.41c)$$

$$\dot{\psi} = D \underline{e}^{ag} \underline{u}_m^T R^{-1} \quad (3.41d)$$

$$P, Q, R > 0$$

Since G_p^a is well defined, G_p is no longer needed for computing D .

3.1.3 Implementation Considerations

3.1.3.1 Actuator Dynamics

As stated in Section 2.1.2 the computed control signal $u(k)$ is in reality applied to a mechanical actuator represented by the dynamics of eqs. 2.9 and 2.10. For the system being considered, the time constants of the primary actuators were less than .08 sec, and the time constants of the secondary actuators were less than .02 sec. These figures are useful in determining the sampling frequency and in ascertaining the importance of these dynamics to the design. Since little would be gained by computing a new control input faster than the actuator can respond, the control sample period should in general be no less than 1/10 the smallest actuator time constant. Thus, it should be expected that control commands applied more frequently will be subjected to actuator filtering.

With regard to the anticipated effect of the actuators upon the behavior of the closed loop system, it should be noted from the actuator data given in Section 2.1.2 and the model data presented in 2.1.5, that the two dominant lateral actuator time constants of 1/30 and 1/25 are significantly less than the time constants of 1/10 and 1/.7 which correspond to the eigenvalues of the lateral model. Similarly the time constant of 1/12.5 associated with the elevator actuator is considerably less than the dominant longitudinal time constant of 1/1.1, and the thrust actuator time constant of 1/5 is less than the velocity time constant of 1/1. Consequently, in the present of good model following, it is anticipated that the effect of the actuators will be negligible relative to the closed loop dynamics. This in fact has been observed from simulations which evaluate the influence of the actuator dynamics in the overall adaptive system.

3.1.3.2 Stability

Although the explicit adaptive control algorithms discussed in Sections 3.1.1.1 and 3.1.1.2 were designed to be stable, it should be noted that this can be guaranteed only if the parameter identification is accurate. If however, at some time, the identification is so poor that the corresponding control gains result in an unstable closed loop system, then the input and output signals will become very oscillatory and large relative to the measurement noise. With this behavior the parameter estimates should quickly converge towards their true values, resulting in a stabilizing effect on the system states. Consequently although an analytical proof is not possible, it can be expected that while the explicit adaptive controllers may at times have rather large outputs, these will be reduced quite rapidly through the resulting improvements in identification.

With regard to stability of the implicit adaptive controller, it should be realized that this was designed, independent of the parameter estimates, for stability in the sense of boundedness of the error between plant and model. Clearly the size of this error will be a function of the initial control gain, the constants in the gain adjustment logic, and the amount of system excitation.

3.1.3.3 Nonlinear Effects

As stated in Section 2.1.1.2, it is necessary to realize that the linear controller algorithms have in reality been designed for regulating incremental motion about the trim states. Thus, it is to be expected that performance will be best for small stepwise perturbations about a constant trim state. For large and possibly time varying pilot commands, it is necessary to compensate for the resulting large incremental motion about trim and also for the effects of changing trim states.

This is partially accomplished by the use of an online identifier which yields those parameters which define the incremental aircraft motion about trim. Also the use of explicit model following, wherein the model trajectory is always used in the controller, tends to compensate for misalignments between plant and model due to uncertainties. More specifically, consideration must be given to the following situations for which linearized model following design is not directly applicable:

- . No pilot command; large external disturbance. - In this situation the trim states should not be altered by the washout filters, and control actions should be such as to return the aircraft to trim. Although the linearized equation will not be indicative of the actual behavior, the estimated parameters should be usable for computing those control actions which will at least reduce the disturbance effects to the point at which linearized analysis will again be valid.
- . Large pilot command; no net change in trim (possible for certain lateral commands). - The effects of a large pilot input, which results in a large deviation about the trim state, might be compensated by the identifier and the feed-forward of the model state. Alternately, such a command might be handled by linearizing the equations (at time k) about the previous states and controls (at time $k-1$). Thus, the control action would be designed so as to make the per sample change in aircraft motion follow the per sample change in the model states. Note that for a step pilot command, the per sample change in $\underline{u}_m(k)$ would be zero for $k > 1$.
- . Large pilot command resulting in a trim variation. - If the pilot command results in a trim variation, then it is necessary to use washout filters with short enough time constants for tracking these variations. Thus, with increasing time, the incremental variables should decrease, in turn making the linearization more accurate.

- Variable pilot command. - If over some extended period, the pilot command is continuously variable, then it may be desirable to have the computer periodically re-initialize the model states to those of the aircraft in order to prevent large errors which would degrade the control effectiveness. This could be performed with a period equal to twice the largest closed loop time constant.

3.1.3.4 Performance Index Weighting Factor Selection

The performance indices (eqs. 3.3a and 3.21a) for the explicit adaptive controllers both contain weighting matrices Q and R which penalize respectively undesirable state trajectories and excessive control signals. These were chosen experimentally according to the following considerations:

- Good following of the two lateral states p and β and of at least two of the longitudinal states q , V , and α .
- Constraints on the control signals -
 $-27.6^\circ < \delta_e < 6.5^\circ$; $|\delta_a| < 30^\circ$, $|\delta_r| < 6^\circ$
- Restriction of the gain values to be such that instabilities do not occur because of actuator nonlinearities.²⁶
- A constant pair of weights Q , R which can be used over the entire flight envelope.

One approach towards the selection of Q and R was to weight only those states of interest and then through a series of simulations decide on the proper ratios. As an example, this procedure resulted in the selection:

$$Q_1 = \begin{pmatrix} 10^4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 10^4 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad R_1 = \begin{pmatrix} 10^2 & 0 \\ 0 & 10^2 \end{pmatrix}$$

for lateral motion and

$$Q_2 = \begin{pmatrix} 100. & 0 & 0 & 0 \\ 0 & 1. & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad R_2 = \begin{pmatrix} 100 \\ 100 \end{pmatrix}$$

for longitudinal motion.

An alternate approach found to be equally effective was to restrict Q and R to again be diagonal with elements:

$$q_{ii} = [\text{maximum } x_i^2]^{-1} p_i^{-2}$$

$$r_{ii} = [\text{maximum } u_i^2]^{-1}$$

where p_i is a weight used to indicate tolerable percentage errors in model following. For lateral motion, the maximum assumed values for p , r , β , ϕ , δ_a , δ_r were respectively 6.98 r/s, .873 r/s, .349r, ∞ , .873r, .873r (i.e., $\delta_{e\text{MAX}} = \delta_{r\text{MAX}}$), for $p_1 = .01$, $p_2 = .10$, and $p_3 = .01$,

$$Q_3 = \begin{pmatrix} 200. & 0 & 0 & 0 \\ 0 & 131. & 0 & 0 \\ 0 & 0 & 82000. & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad R_3 = \begin{pmatrix} 1.31 & 0 \\ 0 & 1.31 \end{pmatrix}$$

For longitudinal motion the maximum assumed values for q , V , α , θ , δ_e , δ_T were respectively 1.75 r/s, 2000 f/s, .524r, ∞ , .873r, 100%. Then for, $p_1 = .01$, $p_2 = .01$, and $p_3 = .10$,

$$Q_4 = \begin{pmatrix} 3280. & 0 & 0 & 0 \\ 0 & .0025 & 0 & 0 \\ 0 & 0 & 364. & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad R_4 = \begin{pmatrix} 1.31 & 0 \\ 0 & .01 \end{pmatrix}$$

In addition to studying the relationship between Q and R and model following errors in the four lateral and four longitudinal states, it was also important to consider the behavior of the vertical acceleration

$$n_z = \frac{V}{g} (\dot{q} - \dot{\alpha})$$

This can indirectly be considered by a judicious choice of the weights on q and α . Alternately n_z can be expressed as a linear transformation of the state variables themselves, and then included in the performance index. Note, however, that since n_z and V are functions of $\dot{\alpha}$, the corresponding transformation will vary with flight condition changes.

Examination of results obtained by penalizing n_z and V simultaneously, revealed that the resulting improvements in n_z behavior were not significant enough to warrant its inclusion into the performance index.

3.2 Estimation Procedures

In designing a procedure for state and parameter estimation, it should be noted that algorithms can be classified according to whether or not states and parameters are estimated simultaneously or separately. Whereas the problem of simultaneous estimation is nonlinear because of the need to determine quantities that multiply each other (parameter times state), the procedures for separately estimating states and/or parameters are linear.

To this effect, following a formal statement of the estimation problem, Sections 3.2.2 and 3.2.3 discuss linear procedures which are used when state estimation is performed separately from parameter identification. Following this presentation, Section 3.2.4 discusses the extended Kalman filter which can be used for simultaneous estimation of both states and parameters. Finally, Sections 3.2.5 and 3.2.6 respectively compare and discuss the implementability of the proposed algorithms.

3.2.1 Problem Statement

The problem considered is that of determining the values of certain parameters appearing in the discretized aircraft equations of motions given exact measurements of the inputs and noisy measurements of the outputs. As given in eq. 2.15, the lateral or longitudinal motion of the aircraft can be represented by the vector difference equation

$$\underline{x}_p(k+1) = A_p(\underline{q}) \underline{x}_p(k) + B_p(\underline{q}) \underline{u}_p(k) \quad (3.42)$$

where \underline{q} is now used to denote a vector whose elements are unknown parameters appearing in the discrete plant matrices. For estimation purposes, it will be assumed that the system measurements can be described by

$$\underline{y}_p(k) = H \underline{x}_p(k) + \underline{n}(k) \quad (3.43)$$

where: $\underline{n}(k)$ is a vector of independent correlated noise sequences with statistics as defined in Appendix B.

and H is a selector matrix indicating just which states or combinations of states are measured. For the problem considered, all states were assumed measureable, and hence $H = I$, the identity matrix.

3.2.2 Weighted Least Squares Identification

Since as indicated by eq. 3.43 the state measurements are corrupted by measurement noise, both state and parameter estimates are needed, the former for control computation and the latter for the gain computation. If the parameters were known, state estimation could be performed using a linear filter derived by minimizing a conventional weighted least squares performance index. Similarly if the states were available, parameter estimates could also be obtained using a linear weighted least squares estimator.

This leads to a two-step procedure at each sample time i ; namely:

Step 1) An estimate for the parameter vector $\underline{q}(i)$ is computed using the measured values $\underline{y}(i)$ and $\underline{y}(i-1)$ for $\underline{x}_p(i)$ and $\underline{x}_p(i-1)$ respectively.

Step 2) An estimate for the state vector $\underline{x}_p(i)$ is computed based upon the matrices $\underline{A}_p(\hat{\underline{q}}(i))$, $\underline{B}_p(\hat{\underline{q}}(i))$.

An alternate procedure for step 1) in which the estimates $\hat{\underline{x}}_p(i)$ and $\hat{\underline{x}}_p(i-1)$, rather than the measurements were used, was observed to yield poor performance. This result was not surprising since large errors in $\hat{\underline{x}}_p$ due to initialization and the subsequent transient response of the identifier, can in turn cause large errors in \underline{q} . The proposed approach however as outlined in step 1) and developed below is completely independent of errors arising from state estimation. In fact, a similar procedure was shown by Anderson et. al, for stable open loop systems, to be consistent for both state and parameter estimation.²⁷

For identification purposes, the time-varying aircraft parameter changes were modelled as fictitious noise disturbances according to the difference equation:

$$\underline{q}(i) = \underline{q}(i-1) + \underline{w}(i-1) \quad (3.44)$$

where \underline{w} is a white Gaussian noise vector with zero mean and covariance matrix \underline{W} .

Eq. 2.1 can then be rewritten in terms of $\underline{q}(k)$ as:

$$\underline{x}_p(i+1) = \underline{C}(i) \underline{q}(i) + \underline{D}(i) \underline{s} \quad (3.45)$$

where \underline{q} and \underline{s} are the vectors containing the unknown and the known elements respectively of the \underline{A} and \underline{B} matrices, and $\underline{C}(k)$ and $\underline{D}(k)$ contain the appropriate measured control^p and state values. A pseudo-measurement vector \underline{z} is then defined as:

$$\underline{z}(i) = \underline{y}_p(i) - \underline{D}(i-1) \underline{s} \quad (3.46)$$

where $\underline{y}_p(i)$ is as defined in eq. 3.43. Estimates $\hat{\underline{q}}(k)$ for the unknown parameter vector $\underline{q}(k)$ can now be determined by forming \underline{C} and \underline{D} from the available measurements and minimizing:²⁸

$$J = \sum_{i=0}^K (\underline{z}(i+1) - \underline{C}(i) \underline{q}(i)) \quad \underline{N}^{-1} (\underline{z}(i+1) - \underline{C}(i) \underline{q}(i)) \quad (3.47)$$

where \underline{N} is an apriori chosen weighting matrix.

The resulting estimates are then defined by:²⁸

$$\hat{\underline{q}}(k+1) = \hat{\underline{q}}(k) + K_q(k+1) (\underline{z}(k+1) - C(k) \hat{\underline{q}}(k)) \quad (3.48)$$

$$K_q(k+1) = P_q(k+1/k) C^T(k) [N + C(k) P_q(k+1/k) C^T(k)]^{-1} \quad (3.49)$$

$$P_q(k+1/k) = P_q(k) + W \quad (3.50)$$

$$P_q(k+1) = P_q(k+1/k) - K_q(k+1) C(k) P_q(k+1/k) \quad (3.51)$$

Note that the fictitious noise covariance matrix W keeps P_q , the parameter covariance matrix, from getting so small that the parameter updating becomes insignificant.

As defined, this identification procedure will yield biased estimates because of the statistical dependence between the noise in $\underline{z}(i)$, $i = 1, \dots, K$ and the components of $C(i)$, $i = 1, \dots, K$.²⁹

For state estimation as required in Step 2), the index

$$J = \sum_{i=0}^K (\underline{y}(i) - \underline{x}_p(i))^T R^{-1} (\underline{y}(i) - \underline{x}_p(i)) \quad (3.52)$$

is minimized subject to:

$$\underline{x}_p(i+1) = A_p(\hat{\underline{q}}(i)) \underline{x}_p(i) + B_p(\hat{\underline{q}}(i)) \underline{u}(i) \quad (3.53)$$

giving:²⁸

$$\hat{\underline{x}}_p(k+1/k) = A_p(\hat{\underline{q}}(k)) \hat{\underline{x}}_p(k) + B_p(\hat{\underline{q}}(k)) \underline{u}(k) \quad (3.54)$$

$$\hat{\underline{x}}_p(k+1) = \hat{\underline{x}}_p(k+1/k) + K_x(k+1) [\underline{y}(k+1) - \hat{\underline{x}}_p(k+1/k)] \quad (3.55)$$

$$K_x(k+1) = P_x(k+1/k) [R + P_x(k+1/k)]^{-1} \quad (3.56)$$

$$P_x(k+1/k) = A_p^T(\hat{\underline{q}}(k)) P_x(k) A_p(\hat{\underline{q}}(k)) \quad (3.57)$$

$$P_x(k+1) = P_x(k+1/k) - K_x(k+1) P_x(k+1/k) \quad (3.58)$$

3.2.3 Minimum Variance Identification

Whereas the weighted least squares identifier discussed in the previous section, neglected the measurement noise contained in $C(i)$ and $D(i)$ as defined in eq. 3.45, it is desirable to develop an alternate

identifier based upon a minimum variance index. To illustrate this approach, it will be assumed that the measurement noise is uncorrelated, i.e.,

$$\begin{aligned} \mathcal{E} [\underline{n}_i \underline{n}_j^T] &= R \quad \text{if } i = j \\ &= 0 \quad \text{if } i \neq j \end{aligned} \quad (3.59)$$

and that the parameter \underline{q} is deterministic and constant, i.e.,

$$\underline{q}(k+1) = \underline{q}(k) \quad (3.60)$$

As in the previous section, a linear identifier will be developed. However, identification will now be performed every other sample period so as to alleviate some of the statistical dependency problems which arise because of the multiplicative noise inherent in the product $C(i) \underline{q}(i)$ of eq. 3.45.³⁰

Thus defining the identification algorithm to be:

$$\hat{\underline{q}}(k+1) = \hat{\underline{q}}(k-1) + K_q(k+1) [\underline{z}(k+1) - C(k) \hat{\underline{q}}(k-1)] \quad (3.61)$$

it is necessary to determine K_q so as to minimize:

$$J = \sum_i \mathcal{E} (\underline{q}_i(k) - \hat{\underline{q}}_i(k))^2 \quad (3.62a)$$

which is the trace of the covariance matrix

$$\mathcal{E} [\underline{q}(k) - \hat{\underline{q}}(k)] [\underline{q}(k) - \hat{\underline{q}}(k)]^T \quad (3.62b)$$

This matrix can in a straightforward manner be formed by substituting eq. 3.61 into eq. 3.62b.³⁰ The resulting minimization of J with respect to elements of the gain matrix K_q yields:³⁰

$$K_q(k+1) = P_q(k) C^T(k) (C^T(k) P_q(k) C(k) + V + R_{eq}(k))^{-1} \quad (3.63)$$

$$P_q(k+1) = P_q(k) - K_q(k+1) C(k) P_q(k) \quad (3.64)$$

where

$$V = \mathcal{E} [C(n_k) \underline{q} \underline{q}^T C^T(n_k)] \quad (3.65)$$

$$R_{eq}(k) = R + \sum [D(n_k) S(k) S^T(k) D^T(n_k)] \quad (3.66)$$

and $C(n_k)$ and $D(n_k)$ denote the nondeterministic portions of C and D (as defined in eq. 3.45).

These equations, except for V and the additional term in R_{eq} are identical to eqs. 3.54-3.58 if W is set to equal to zero and the weighting matrix N is replaced by $(V + R_{eq})$.

In practice V can be computed using either the initial estimate or the most recent estimate for \underline{q} .

Although this linear minimum variance algorithm as given by eqs. 3.61-3.66, will still yield biased estimates, it is possible to alter eq. 3.61 so that the estimate for \underline{q} will be at least asymptotically unbiased, i.e.,

$$\lim_{k \rightarrow \infty} \sum (\hat{\underline{q}}_k) = \underline{q}$$

This will be true if the prediction term in eq. 3.61 is modified, resulting in:³⁰

$$\begin{aligned} \hat{\underline{q}}(k+1) = & (I + P_q(k+1) \sum [C^T(n_k) [R_{eq} + V] C(n_k)]^{-1}) \hat{\underline{q}}(k-1) \\ & + K_q(k+1) (\underline{z}(k+1) - C(k) \hat{\underline{q}}(k-1)) \end{aligned} \quad (3.67)$$

In a similar manner, it can be shown that the minimum variance estimator for the states taking into account the parameter uncertainty is defined as follows:³⁰

$$\hat{\underline{x}}_p(k+1/k) = A_p(\underline{q}(k)) \hat{\underline{x}}_p(k) + B_p(\underline{q}(k)) \underline{u}_p(k) \quad (3.68)$$

$$\hat{\underline{x}}_p(k+1) = \hat{\underline{x}}_p(k+1/k) + K_x(k+1) [\underline{y}(k+1) - \hat{\underline{x}}_p(k+1/k)] \quad (3.69)$$

$$K_x(k+1) = P_x(k+1/k) [R + P_x(k+1/k)]^{-1} \quad (3.70)$$

$$\begin{aligned} P_x(k+1/k) = & A_p^T(\hat{\underline{q}}(k)) P_x(k) A_p(\hat{\underline{q}}(k)) \\ & + \sum [C(x(k)) P_q(k) C^T(x(k))] \end{aligned} \quad (3.71)$$

Comparison with the weighted least squares state estimation algorithm eqs. 3.54-3.58 shows that the predicted covariance (eqs. 3.57 and 3.71) is increased by the additional term:

$$\sum [C(x(k)) P_q(k) C^T(x(k))]$$

where P_q is as defined in eq. 3.64.

3.2.4 Extended Kalman Filter

To simultaneously estimate both the states \underline{x}_p , and the unknown parameters \underline{q} , appearing in the state transition matrix A_p and the gain matrix B_p , it is necessary to form an augmented state vector. This is done by appending to the aircraft equations (2.15) the parameter equation 3.44. Then the augmented system becomes:

$$\underline{x}^a(k+1) = A^a(\underline{x}^a(k), k) \underline{x}^a + B(\underline{x}^a(k), k) \underline{u}(k) \quad (3.72)$$

$$\underline{y}(k) = H^a(k) \underline{x}^a(k) + n(k) \quad (3.73)$$

where

\underline{x}^a = augmented state vector, given by

$$\underline{x}^a = \begin{bmatrix} \underline{x}_p \\ \underline{q} \end{bmatrix}$$

$$A^a = \begin{bmatrix} A_p(\underline{q}) & N_1 \\ N_2 & I_p \end{bmatrix}$$

$$B^a = \begin{bmatrix} B_p(\underline{q}) \\ 0 \end{bmatrix} \quad (3.74)$$

and H^a = the augmented system output matrix given by

$$H^a = \begin{bmatrix} H & | & N_3 \end{bmatrix}$$

I_p is an identity matrix of dimension $(p \times p)$, and N_1 , N_2 and N_3 are null matrices with dimensions of $(n \times p)$, $(p \times n)$ and $(n \times p)$ respectively.

To estimate the augmented vector \underline{x}^a using a Kalman filter, it is necessary to linearize eq. 3.72 about some nominal trajectory $(\underline{x}^o, \underline{q}^o)$. Denoting

$$\underline{x}_p^a - \underline{x}_p^o \quad \text{as} \quad \Delta \underline{x}_p \quad (3.75)$$

$$\underline{q} - \underline{q}^o \quad \text{as} \quad \Delta \underline{q} \quad (3.76)$$

$$\text{and} \quad \underline{y}(i) - H \underline{x}_p^o \quad \text{as} \quad \Delta \underline{y}, \quad (3.77)$$

the linearized versions of eqs. 3.72 and 3.73 become:

$$\Delta \underline{x}_p(i+1) = A_p(\underline{q}^o) \Delta \underline{x}_p(i) + \frac{\partial}{\partial \underline{q}} \{A_p(\underline{q}) \underline{x}_p + B_p(\underline{q}) \underline{u}_p\} \Delta \underline{q} \quad (3.78)$$

$$\Delta \underline{q}(i+1) = \Delta \underline{q}(i) \quad (3.79)$$

$$\Delta \underline{y}(i) = H \Delta \underline{x}_p(i) + n(i) \quad (3.80)$$

These equations which are linear in $\Delta \underline{x}_p$ and in $\Delta \underline{q}$ are now amenable to linear weighted least squares estimation of the augmented state $(\Delta \underline{x}_p, \Delta \underline{q})$. If the nominal trajectory $(\underline{x}_p^o, \underline{q}^o)$ is always updated to correspond with the most recent estimate, i.e.,

$$\underline{x}_p^o(i-1) = \hat{\underline{x}}_p(i-1/i-1) \quad (3.81)$$

$$\underline{x}_p^o(i) = A_p(\hat{\underline{q}}(i-1)) \underline{x}_p^o(i-1) + B_p(\hat{\underline{q}}(i-1)) \underline{u}_p(i-1) \quad (3.82)$$

$$\underline{q}^o(i-1) = \hat{\underline{q}}(i-1/i-1) \quad (3.83)$$

$$\underline{q}^o(i) = \underline{q}^o(i-1) \quad (3.84)$$

then the resulting filter equations become:^{18,31,32}

$$\begin{pmatrix} \hat{\underline{x}}_p(i/i) \\ \hat{\underline{q}}(i/i) \end{pmatrix} = \begin{pmatrix} \hat{\underline{x}}_p(i-1/i-1) \\ \hat{\underline{q}}(i-1/i-1) \end{pmatrix} + K(i) (\underline{y}(i) - H \hat{\underline{x}}_p(i-1/i-1)) \quad (3.85)$$

where

$$\hat{x}_p(i/i-1) = A_p(\hat{q}(i-1/i-1)) \hat{x}_p(i-1/i-1) + B_p(\hat{q}(i-1/i-1)) u_p(i-1) \quad (3.86)$$

$$K(i) = P(i/i-1) H^a(i)^T [H^a(i) P(i/i-1) H^T(i) + R]^{-1} \quad (3.87)$$

$$P(i/i-1) = J(i) P(i-1/i-1) J^T(i) \quad (3.88)$$

$$P(i/i) = P(i/i-1) - K(i) H^a(i-1) P(i/i-1) \quad (3.89)$$

and

$$J(i) = \begin{bmatrix} A_p(q(i)) & \frac{\partial}{\partial q} [A_p(q) x_p + B_p(q) u_p] \\ 0 & I \end{bmatrix} \begin{matrix} x_p^0(i) \\ q^0(i) \end{matrix} \quad (3.90)$$

With regard to the convergence of this algorithm, it can be shown that if the initial errors between the estimated and the true values are sufficiently large, the estimates as defined by eqs. 3.85-3.90 will diverge.³⁰ This behavior follows from neglecting the nonlinearities which can propagate through the computations as systematic noise. Such behavior was in fact observed in several computer simulations.

3.2.5 Comparative Discussion

Taking into account convergence properties, requirements for implementation, and observed performance in simulation experiments, it is recommended that the weighted least squares algorithm as discussed in Section 3.2.2 be utilized.

Although as shown in Section 3.2.3, the minimum variance procedures do take into account the system noise and at the same time can be implemented almost as easily as the weighted least squares procedures, it was noted during simulation, that the minimum variance state estimates were not accurate enough for effective control computation. This is the result of including in the computation of P_q (eq. 3.70) the parameter uncertainty P_x . Thus a large initial value of $x P_q$ (which is needed for rapid convergence of the parameter estimates) can immediately cause the state estimates to track the measurement noise. However, it should be noted that the parameter estimates obtained from the minimum variance procedures were significantly better than those resulting from either the extended Kalman filter or the weighted least squares procedures.

The extended Kalman filter is not recommended for implementation primarily because of its divergent properties in the presence of large parameter errors³⁰ and secondarily because of its relative complexity. Since it is possible for the aircraft to change flight conditions without sufficient excitation for identification, it is indeed probable that in the presence of a pilot command, the parameter errors will be large enough such that divergence will occur. Such large errors have in fact been observed in simulation experiments.

3.2.6 Implementation Considerations

3.2.6.1 Identifiability

In order for the parameter estimates to be meaningful, it is necessary that the overall structure and excitation be such that the system is identifiable. For the problem defined by eqs. 3.46 and 3.43, it has been shown that³²

- . If the only parameters to be identified are elements of A_p , then no restrictions are needed.
- . If elements of B_p are to be identified then the control inputs u_{p_i} must be independent.

In addition to these necessary conditions, it is also desirable to consider control inputs for optimizing the performance of the identifier. In particular, if these inputs are to be found so as to maximize the weighted trace of the Fisher information matrix, then it can be shown that the optimal energy constrained input can be defined by the eigenvalues and eigenfunctions of a two point boundary value problem.^{33,34} However, because of the complex nature of this problem and the need to restrict external inputs so as to be imperceptible to the pilot, such an approach was not pursued.

Thus, as sources for sufficient excitation of the identifier, only the following were considered:

- . On-off type dither inputs (discussed in 3.26).
- . Pilot commands.
- . Large and oscillatory motions resulting from control gains computed from poor parameter estimates.

3.2.6.2 Influence of Inputs Other Than Pilot Commands

Implementation of an online identifier requires that consideration be given not only to behavior in the presence of pilot commands, but also to the behavior resulting from dither, gusts, and sensor noise.

To assess the effects of dither, square wave signals with random switching times were studied for their utility in improving the identification especially during periods of steady flight motion. However, with the

restriction that the switching frequency and the dither magnitude be such that the resulting effects be imperceptible to the pilot, the resulting dither signals were found to be non-influential in the noise environment defined in Appendix B. Despite this, it was noted that the feedback of the filtered measurement noise itself did offer some improvements in parameter tracking.

Although no significant dither could be applied to the aircraft during a steady transition between flight conditions, it was conjectured, and shown by simulation, that if the aircraft changes flight conditions without proper tracking by the identifier, then in response to a pilot command, the aircraft may undergo large oscillatory motion due to improper control gains. This would then have the effect of exciting the identifier enough to result in rapid estimation of the proper parameter values. Such behavior was indeed observed even when the initial parameters were such as to produce control gains which destabilized the system. Oscillations were in these cases observed to be eliminated within seven seconds.

The effects of gust disturbances can be assessed by reconsidering the longitudinal equations modified as in eq. 2.14 to account for gusts:

$$\dot{\underline{x}}_p = A_p \underline{x}_p + A_p (3) \Delta \alpha + B_p \underline{u}_p \quad (3.91)$$

Thus if a wind sensor is used so that $\Delta \alpha$ is available, then the gust can be regarded as an additional excitation aiding the identification. However, it is anticipated that the effects of such gusts will be dampened out since the controller is designed so as to reduce the error between plant and model (which is itself not excited by the gust).

If the gust disturbance $\Delta \alpha$ is not measured then it should be modelled as a process noise term added to the aircraft equation.²⁸

Concerning sensor noise, it was observed that the specifications cited in Appendix B were such that the response contained some oscillations due to the filter's inability to completely smooth out the included effects of the bending modes. A reduction of the noise levels to one tenth of the Appendix B values did however result in significant improvements. Although the estimation algorithms considered did not take into account the correlated nature of the sensor noise, it was observed that the simulation of uncorrelated measurement noise sequences did not result in any noticeable improvement.

3.2.6.3 Initialization

In designing estimation logic for an adaptive controller, it is necessary to determine initial values for the state and parameter estimates, the corresponding covariance matrices, and the parameters W and R as defined in eq. 3.44 and eq. 3.52 respectively. To this effect the following guidelines have been determined through simulation efforts:

- . Initialize parameters equal to their average values as computed over the flight envelope.
- . Initialize the states equal to the measured values.
- . Select the initial variance of each parameter estimate to be in proportion to the square of three times its largest possible value. A factor of 10^4 times these values was observed to yield satisfactory convergence.
- . Select the initial variance of each state to be zero if the initial state values are well defined.
- . Select R anywhere between one and five times the actual noise covariance matrix.
- . Select the elements of W equal to three times the expected square of the per sample change in each of the identified parameters.

In addition, so that the identification does not become too complex and time consuming, it is desirable to select which parameters need to be identified and which can be allowed to remain at the average value. A sensitivity procedure which uses the state sensitivity vectors $\partial \underline{x}_p / \partial q$ to predict the change $\Delta \underline{x}_p$ in \underline{x}_p resulting from a corresponding parameter change Δq showed that only eight to twelve parameters need be identified for either the longitudinal or the lateral control system.⁶

4. EXPERIMENTAL RESULTS AND DISCUSSION

Evaluation of the proposed digital adaptive flight control systems was based upon a series of simulation experiments performed on Rensselaer's IBM 360/67 computer and on NASA Langley's CDC 6600 computer. These experiments considered the application of the controllers to both the linearized equations of motion (as presented in Section 4.1) and the nonlinear six-degree-of-freedom simulation³⁵ (as presented in Section 4.2).

4.1 Linear System Evaluation

Because of the need to examine the required preciseness of identification and the degree of adaption needed, a typical flight trajectory in the altitude-mach number plane was postulated. This is defined in Appendix C which cites the order and timing for a typical fighter aircraft to encounter the six given flight conditions of Appendix A. This trajectory corresponds to an initial acceleration from Mach .3 to Mach .9 at a very low altitude, a combined climb to 3600 m and acceleration to Mach 1.1, a climb to 15,000 m, a deceleration to Mach .9, and finally a combined dive to 6000 m and a deceleration to Mach .7. For simulation purposes, it was assumed that the parameters of the aircraft's discrete A_p and B_p matrices varied linearly with time between these flight conditions.

As stated in Section 3.2.6.3, it is impractical, for explicit adaptation, to consider the identification of all parameters of the A_p and B_p matrices. A sensitivity study as discussed in reference 6 was therefore performed to determine those parameters which least effected system performance and which might therefore be considered constant. This effect upon system performance considered not only the sensitivity vector but also the possible change in each parameter. Thus even if a particular parameter is highly influential, it could be excluded from identification if its expected variation is negligible.

Results of this study suggested that for lateral motion, the following eleven parameters be set to their average values and thus not be identified:

$$A_p(1,4), A_p(2,1), A_p(2,2), A_p(2,4), A_p(4,1), \\ A_p(4,2), A_p(4,3), A_p(4,4), B_p(2,1), B_p(4,1), B_p(4,2)$$

Additional parameters might be included in this list according to the particular pilot command being applied.

For longitudinal motion, the parameters not identified and therefore frozen at their average values included:

$$A_p(1,2), A_p(1,4), A_p(2,1), A_p(2,2), A_p(2,4), A_p(3,1), \\ A_p(3,2), A_p(3,4), A_p(4,1), A_p(4,2), A_p(4,4), B_p(1,2), \\ B_p(2,1), B_p(2,2), B_p(3,2), B_p(4,2)$$

Prior to testing the explicit adaptive controllers with the identification logic an evaluation of the gain update logic itself was made under the assumption of perfect identification of various influential sets of time varying parameters.⁶ This study, based upon the trajectory defined in Appendix C, resulted in the suggestion that gain adaptation be performed once every second and that the control signal itself be updated every 0.1 to 0.2 seconds.⁶ These results were based upon reponse observation and therefore could be altered by pilot opinion.

4.1.1 Adaptive Optimal Linear Regulator Results

Results pertaining to the performance of the adaptive optimal linear regulator controller applied to the linearized lateral equation of motion may be found in references 6 and 17. Therefore, the following description is concerned only with the behavior of the adaptively controlled linearized longitudinal equations of motion.

Objective:

To study, using linearized longitudinal equations, the behavior of the adaptive optimal linear regulator discussed in Section 3.1.1.1.

Procedure:

Using noisy state measurements, parameter estimates were obtained at each control sample period and then used at each gain update sample in one iteration of the Riccati equation (2.9). The gains were then computed and used for control computation. Performance with and without state estimation was considered.

Constant Factors:

Pilot input: $\pm .1$ radian elevator, 0.2 Hz square wave

Control sample period: 0.1 sec.

Gain adaptation period: 1 sec.

Identifier: weighted least squares (Section 3.22)

Parameters identified: $A_p(1, 1)$, $A_p(1, 3)$, $A_p(2, 3)$, $A_p(3, 3)$,
 $A_p(4, 3)$, $B_p(1, 1)$, $B_p(3, 1)$, $B_p(4, 1)$

Initially set equal to their true values.

Remaining parameters: Set at their average values as computed over the 6 FCS.

Measurement noise: as stated in Appendix B

Control index weights: Q_3 and R_3 as defined in Section 3.1.3.4

Results and Discussion:

Because of the relatively low variance inherent in the longitudinal sensors, the use of a state estimator was observed from the initial simulations to be unnecessary and was therefore omitted in subsequent experiments.

Figures 4.1-4.7 depict the state responses, vertical acceleration, and controls while Figures 4.8-4.10 give the identified values for three of the eight parameters being tracked. Corresponding feedback gains are shown in Figure 4.11.

These figures and observations made beyond the illustrated records, indicated the capability for tracking the gain variations, thus resulting in acceptable V , α , θ , responses. It was further observed that of the eight parameters being identified, six were noted to track reasonably

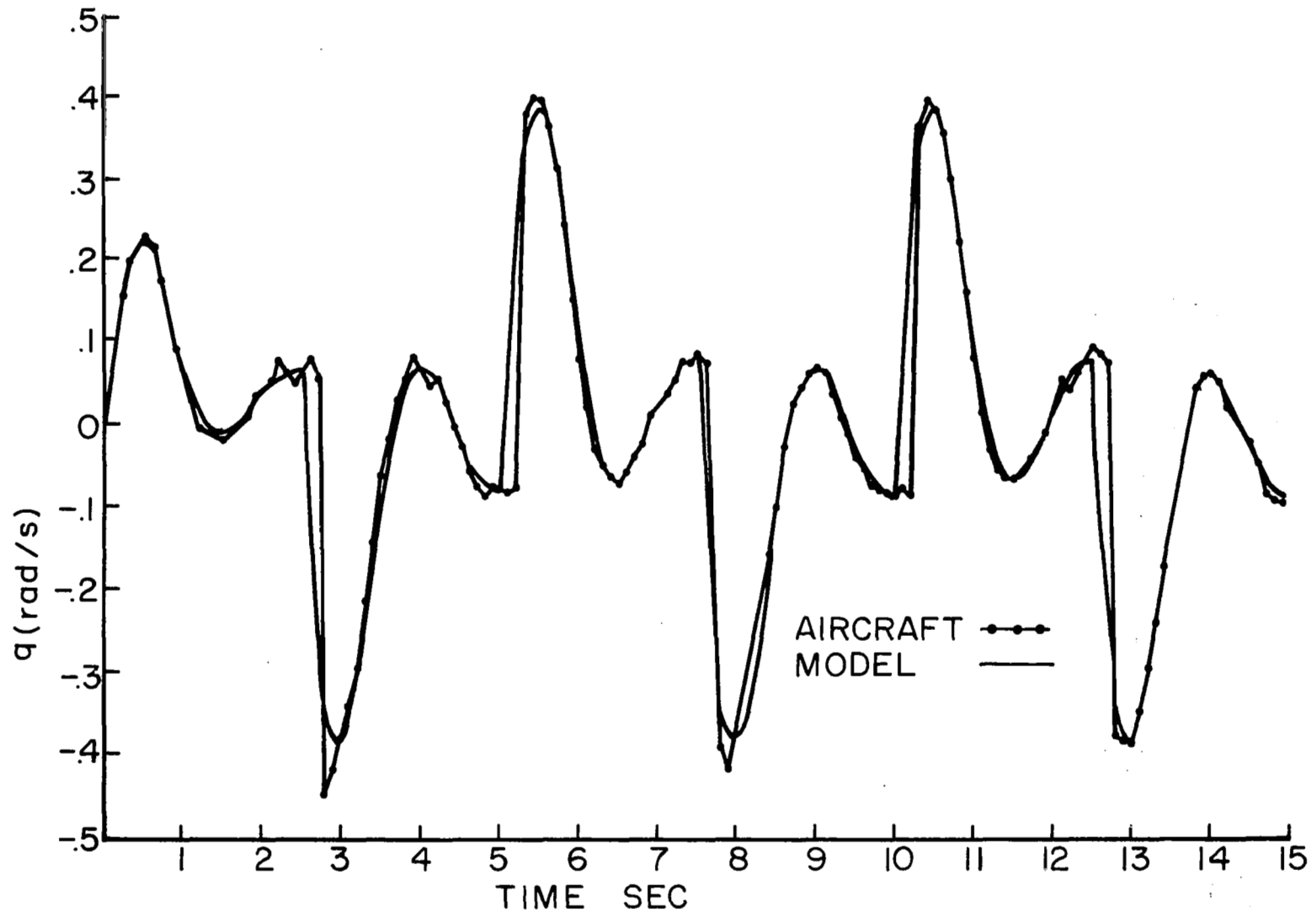


FIG. 4.1 PITCH RATE RESPONSE TO 0.1 RAD, 0.2 Hz ELEVATOR COMMAND, ADAPTIVE OPTIMAL REGULATOR (SECTION 4.1.1)

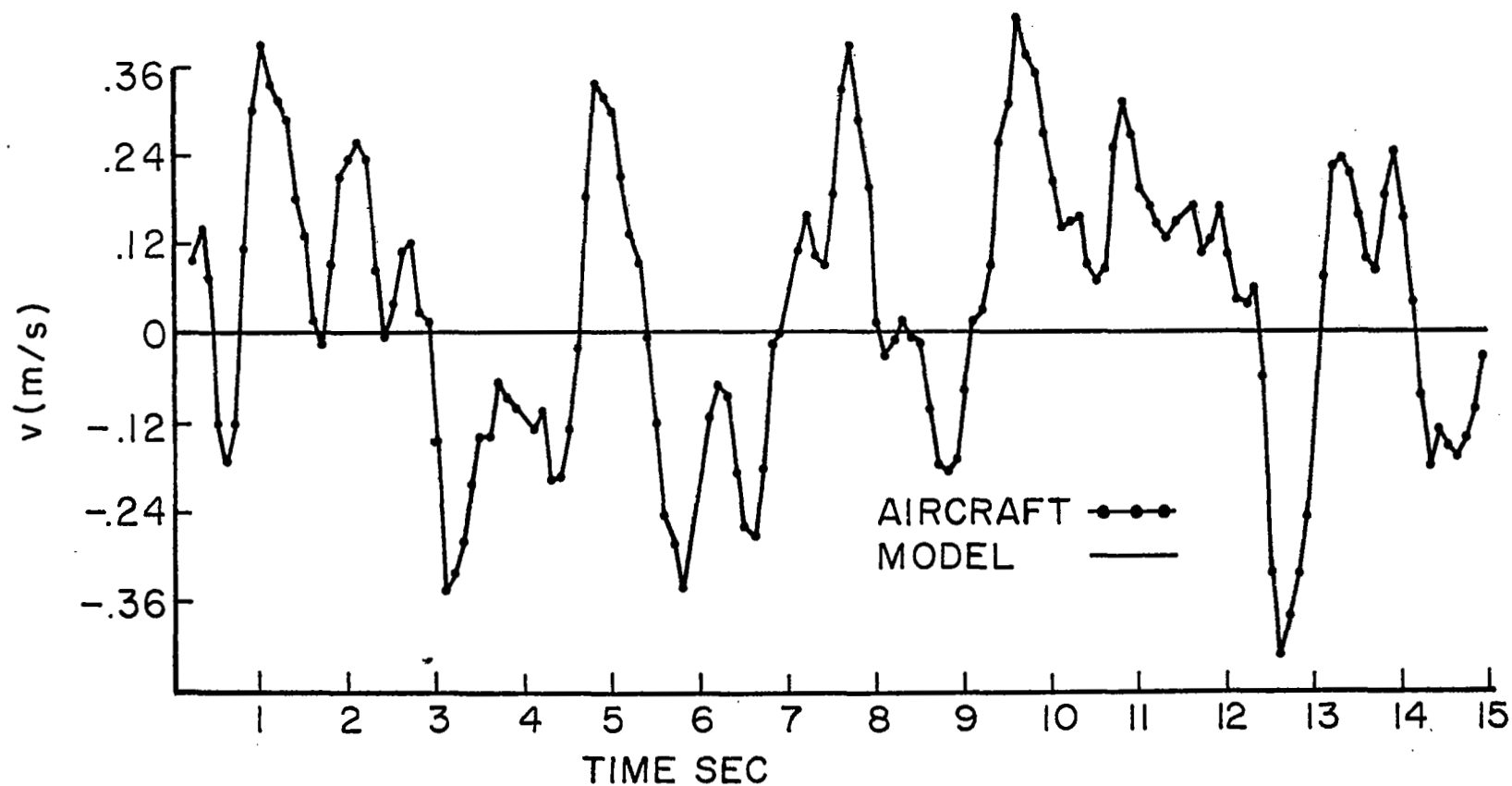


FIG. 4.2 VELOCITY RESPONSE TO 0.1 RAD, 0.2 Hz ELEVATOR COMMAND, ADAPTIVE OPTIMAL REGULATOR (SECTION 4.1.1)

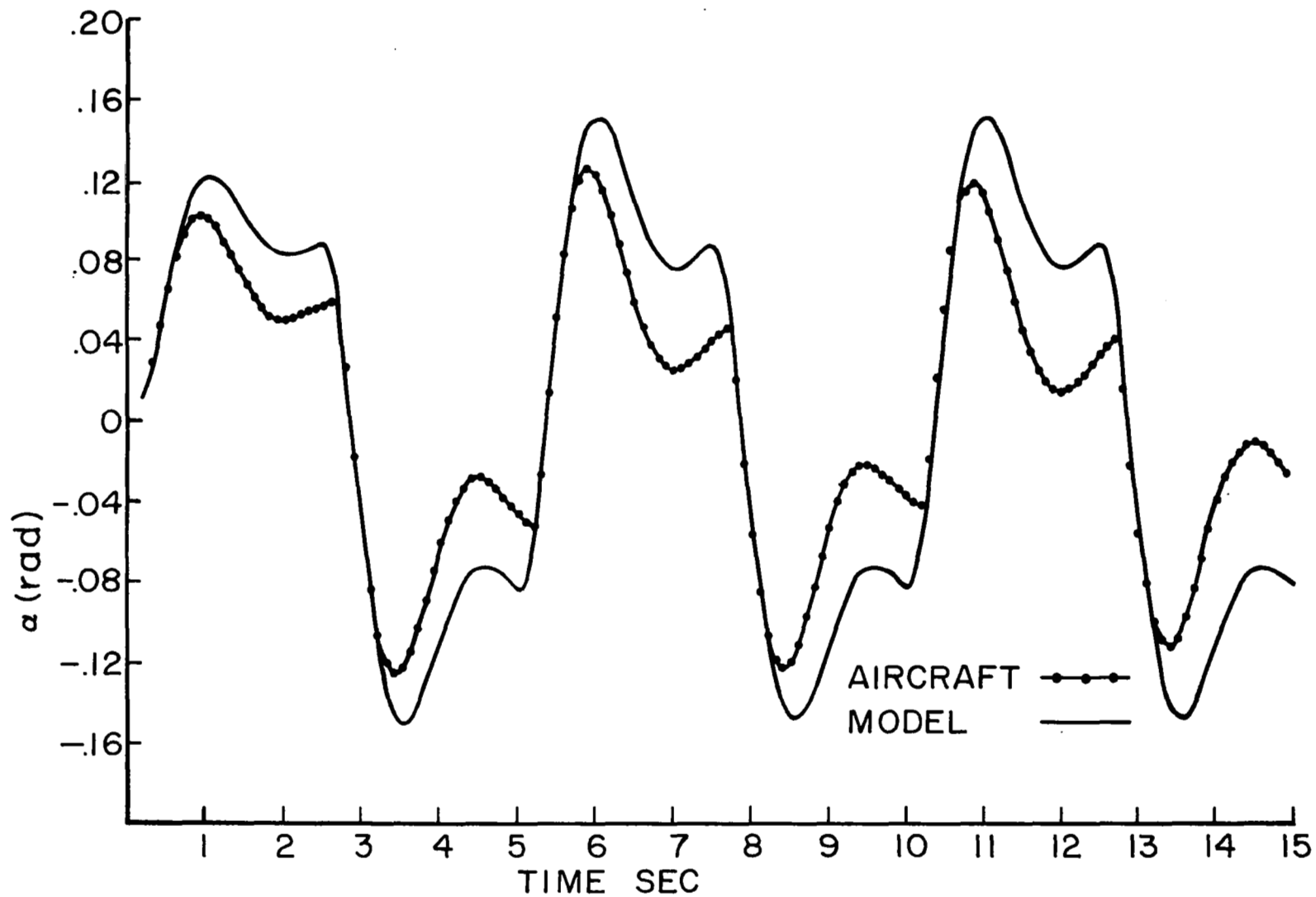


FIG. 4.3 ANGLE OF ATTACK RESPONSE TO 0.1 RAD, 0.2 Hz ELEVATOR COMMAND, ADAPTIVE OPTIMAL REGULATOR (SECTION 4.1.1)

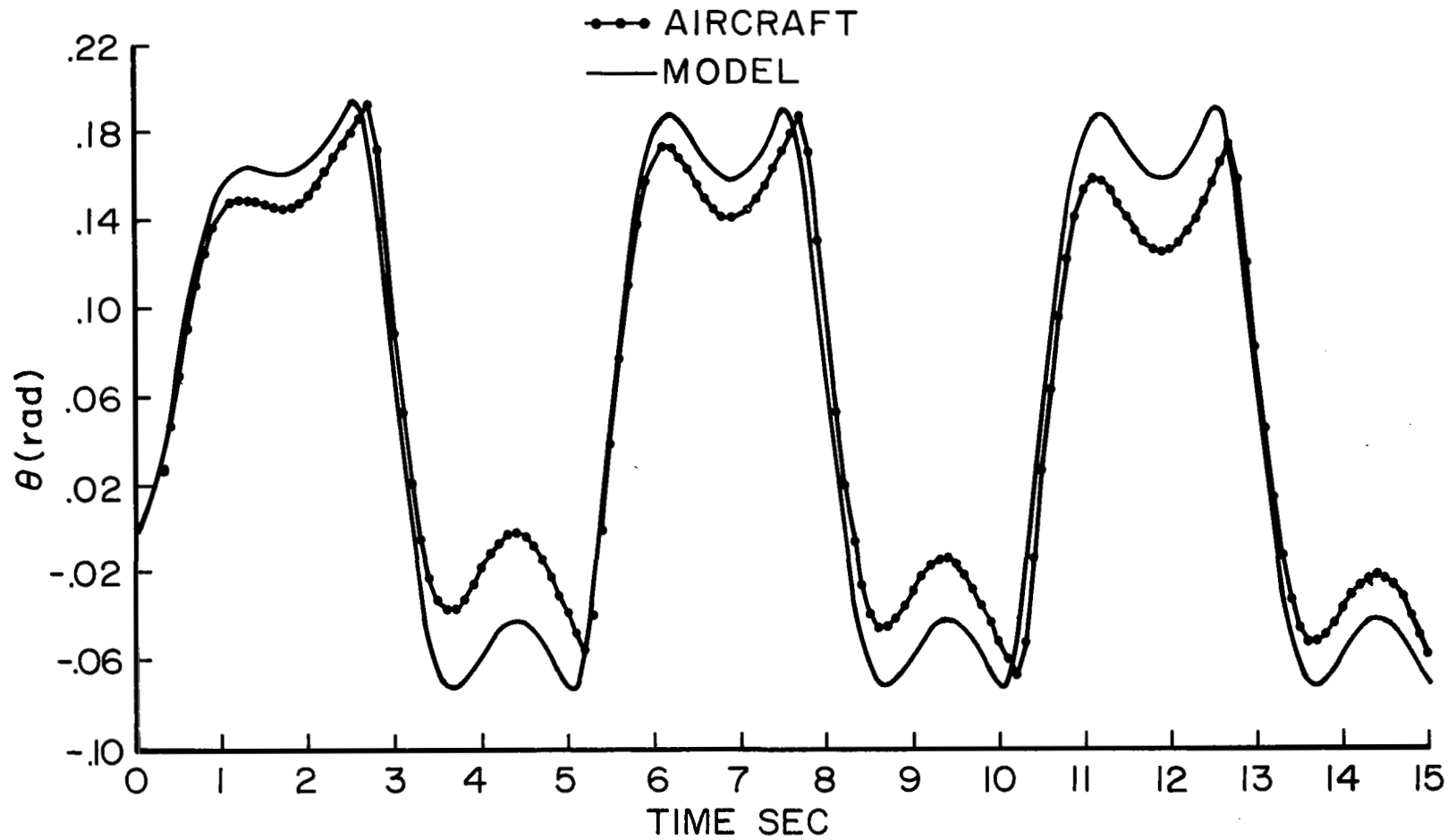


FIG. 4.4 PITCH RESPONSE TO 0.1 RAD 0.2 hz ELEVATOR COMMAND, ADAPTIVE OPTIMAL REGULATOR (SECTION 4.1.1)

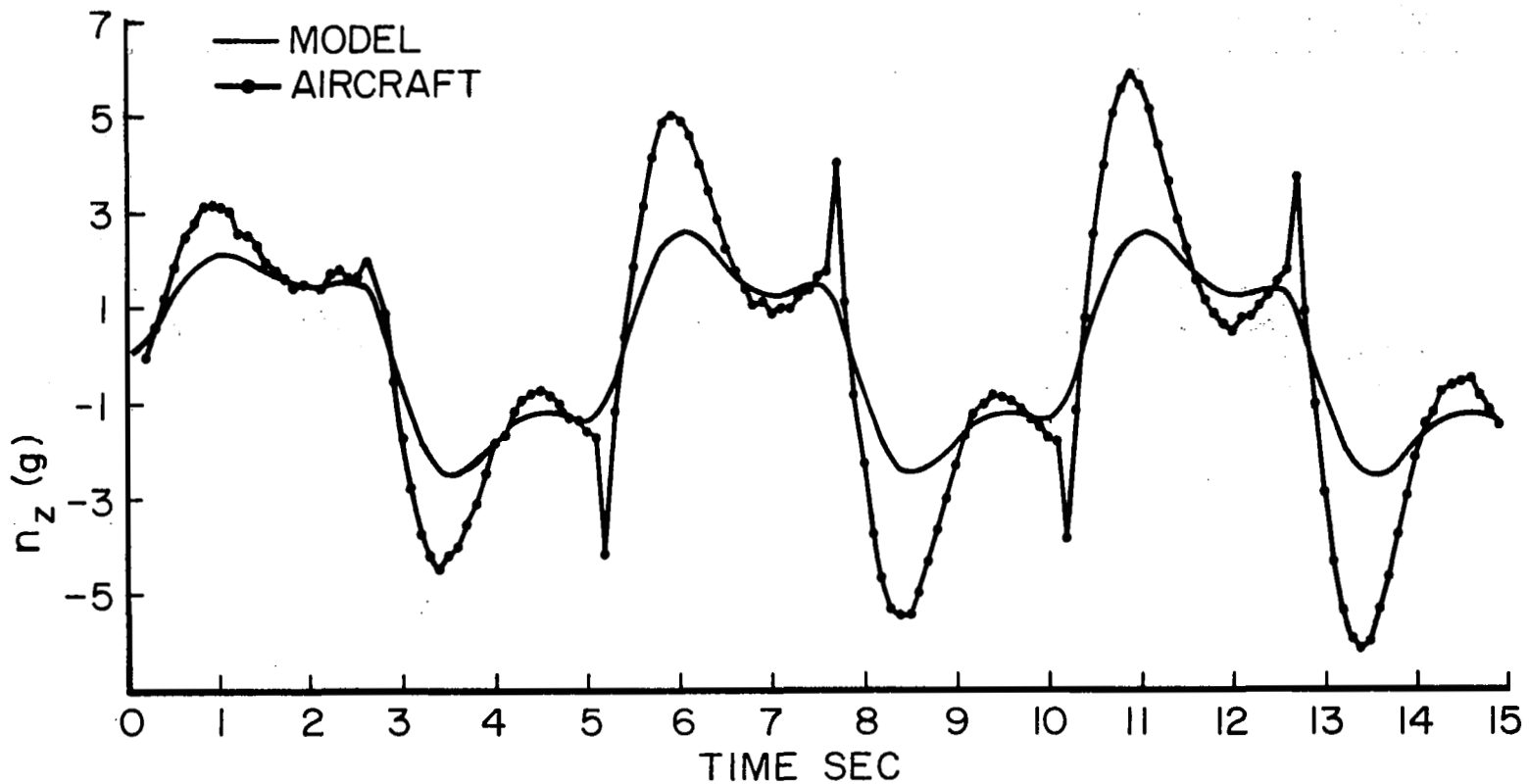


FIG. 4.5 VERTICAL ACCELERATION RESPONSE TO 0.1 RAD 0.2 Hz, ELEVATOR COMMAND, ADAPTIVE OPTIMAL REGULATOR (SECTION 4.1.1)

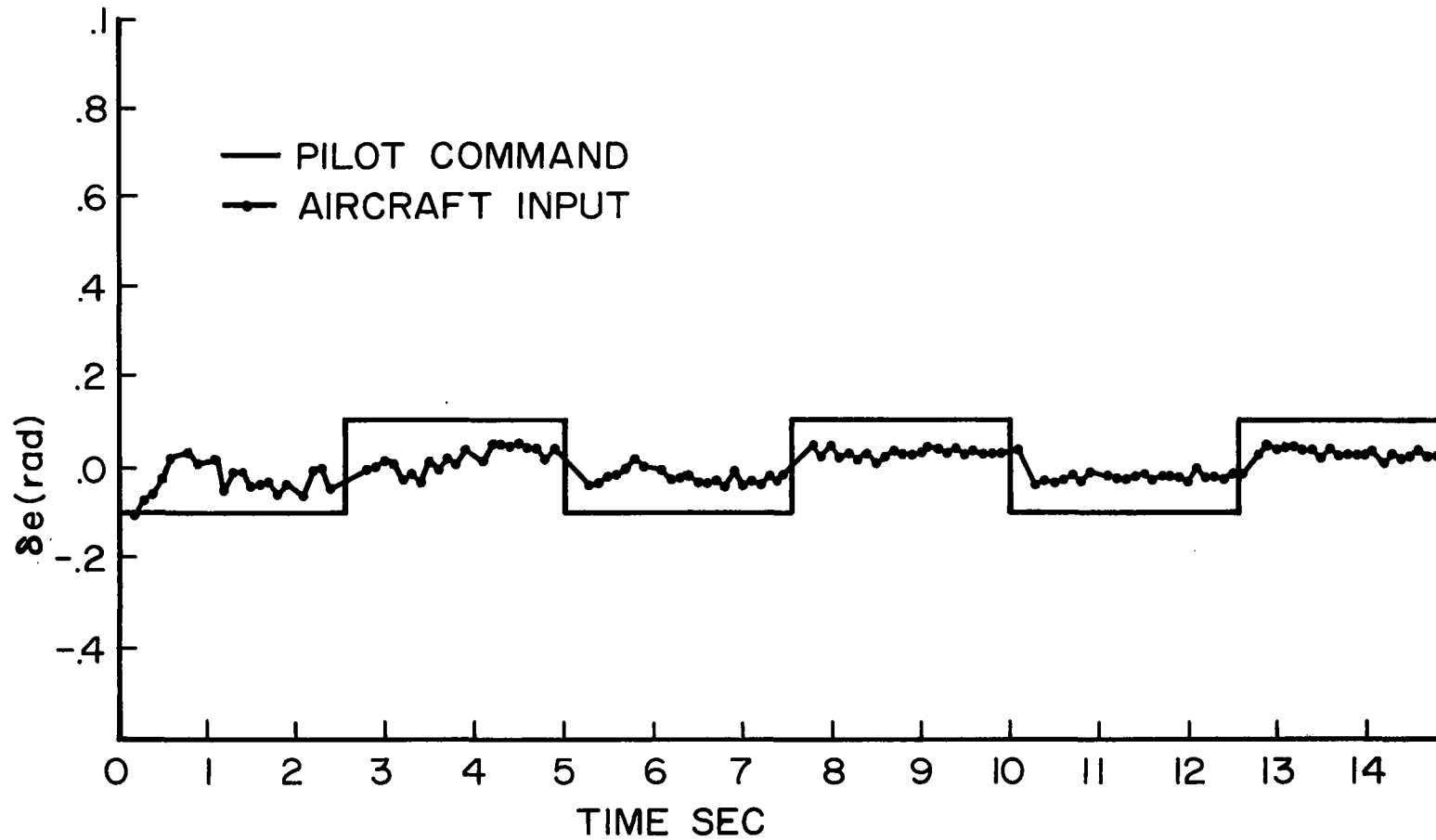


FIG. 4.6 PILOT COMMAND AND AIRCRAFT ELEVATOR INPUT, ADAPTIVE OPTIMAL REGULATOR (SECTION 4.1.1)

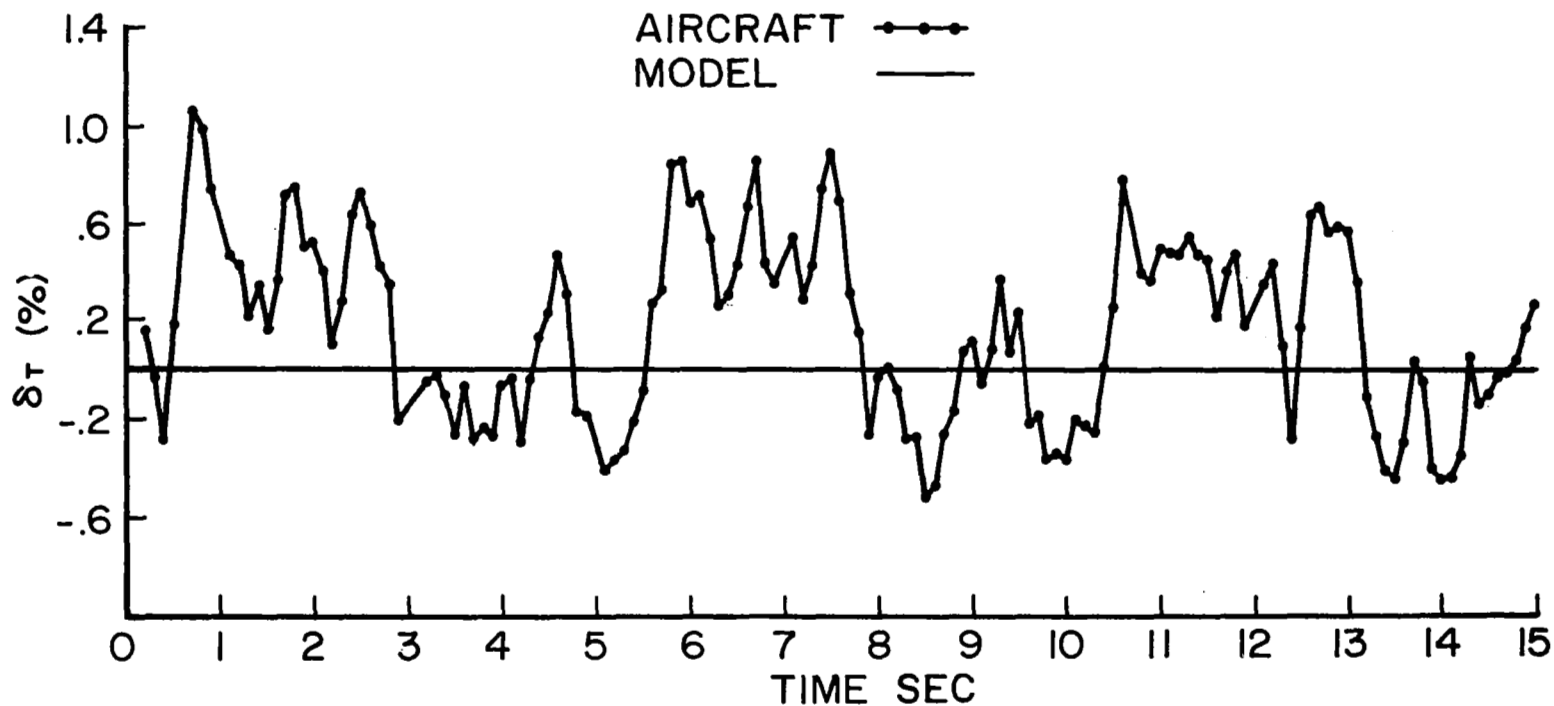


FIG. 4.7 AIRCRAFT THRUST INPUT IN RESPONSE TO A 0.1 RAD, 0.2hz ELEVATOR COMMAND, ADAPTIVE OPTIMAL REGULATOR (SECTION 4.1.1)

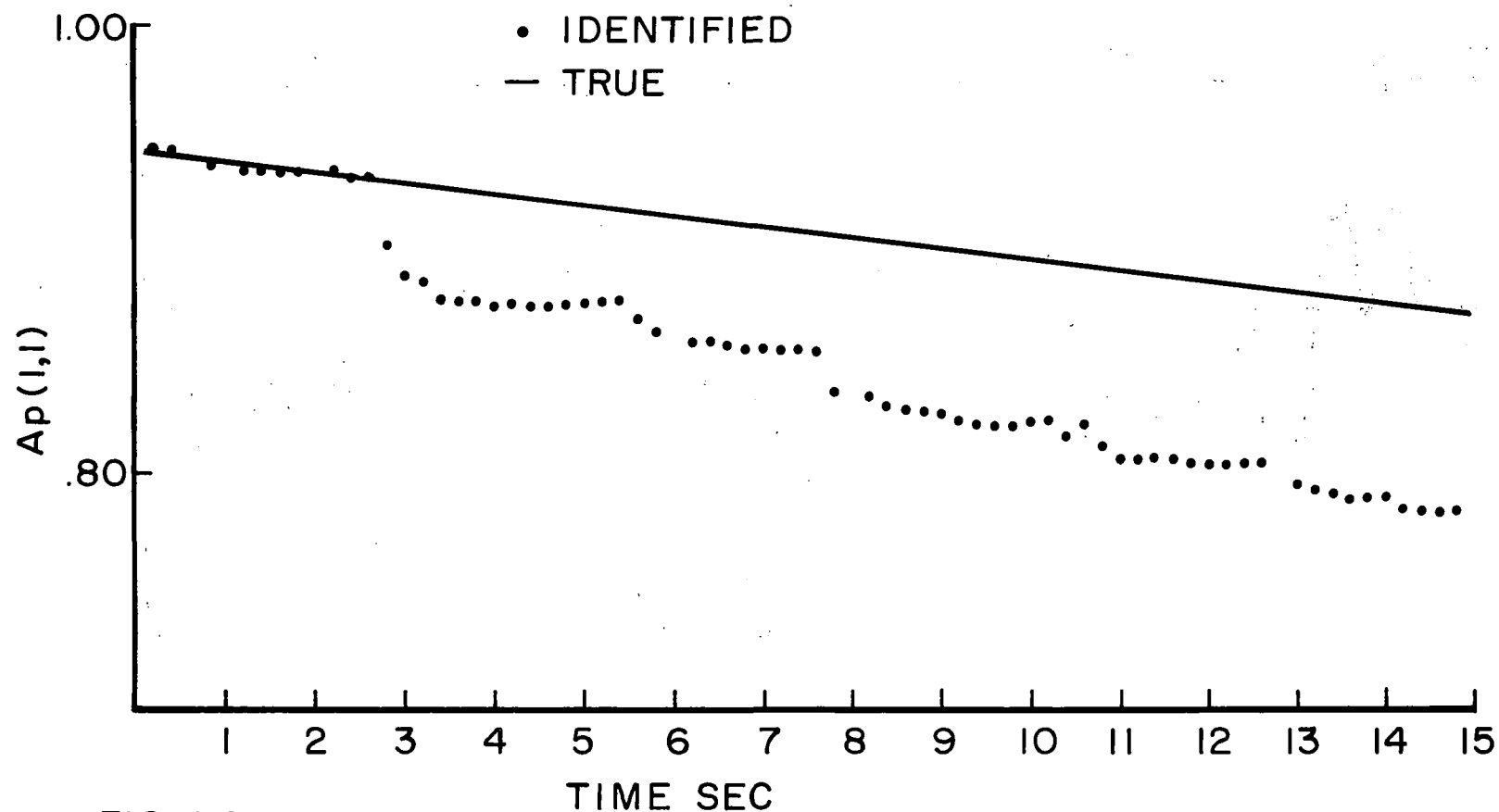


FIG. 4.8
 PARAMETER $A_p(1,1)$ FOR LONGITUDINAL ADAPTIVE OPTIMAL REGULATOR,
 PILOT COMMAND = 0.1 RAD, 0.2 Hz ELEVATOR (SECTION 4.11)

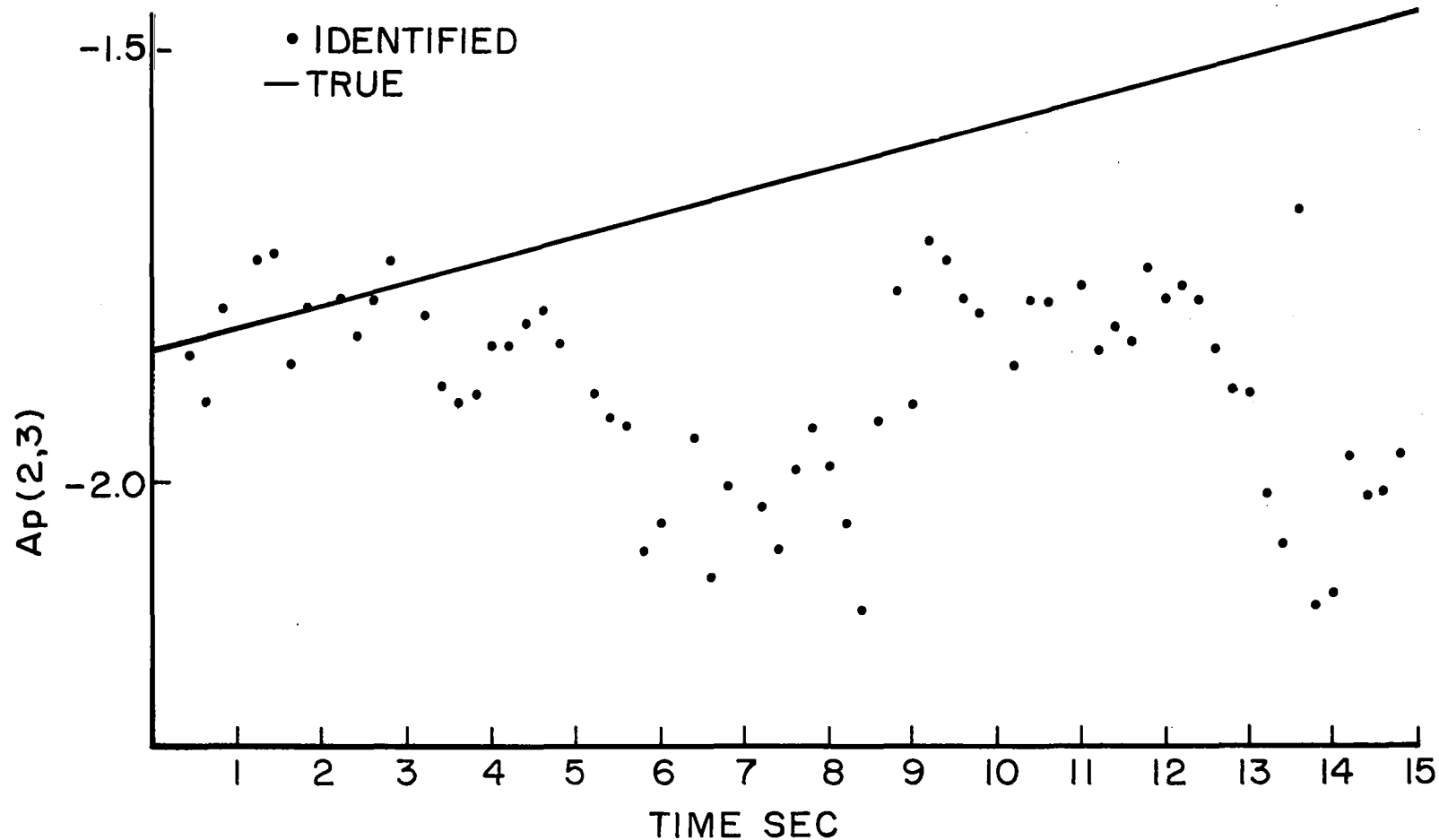


FIG.4.9 PARAMETER $A_p(2,3)$ FOR LONGITUDINAL ADAPTIVE OPTIMAL REGULATOR PILOT COMMAND=0.1 RAD,0.2 hz ELEVATOR, (SECTION 4.1.1)

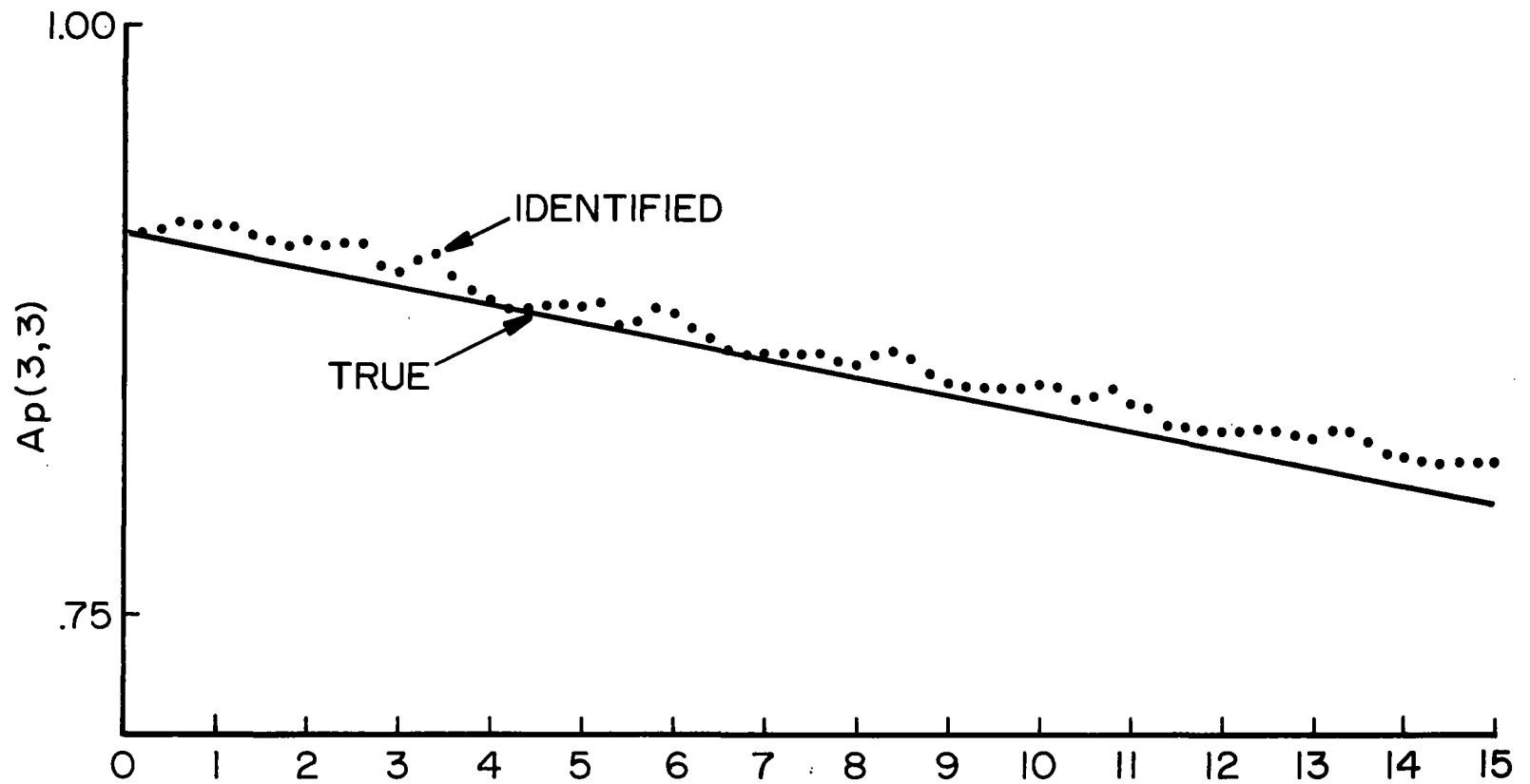


FIG. 4.10 PARAMETER $A_p(3,3)$ FOR LONGITUDINAL ADAPTIVE OPTIMAL REGULATOR, PILOT COMMAND = 0.1 RAD, 0.2 HZ ELEVATOR (SECTION 4.1.1)

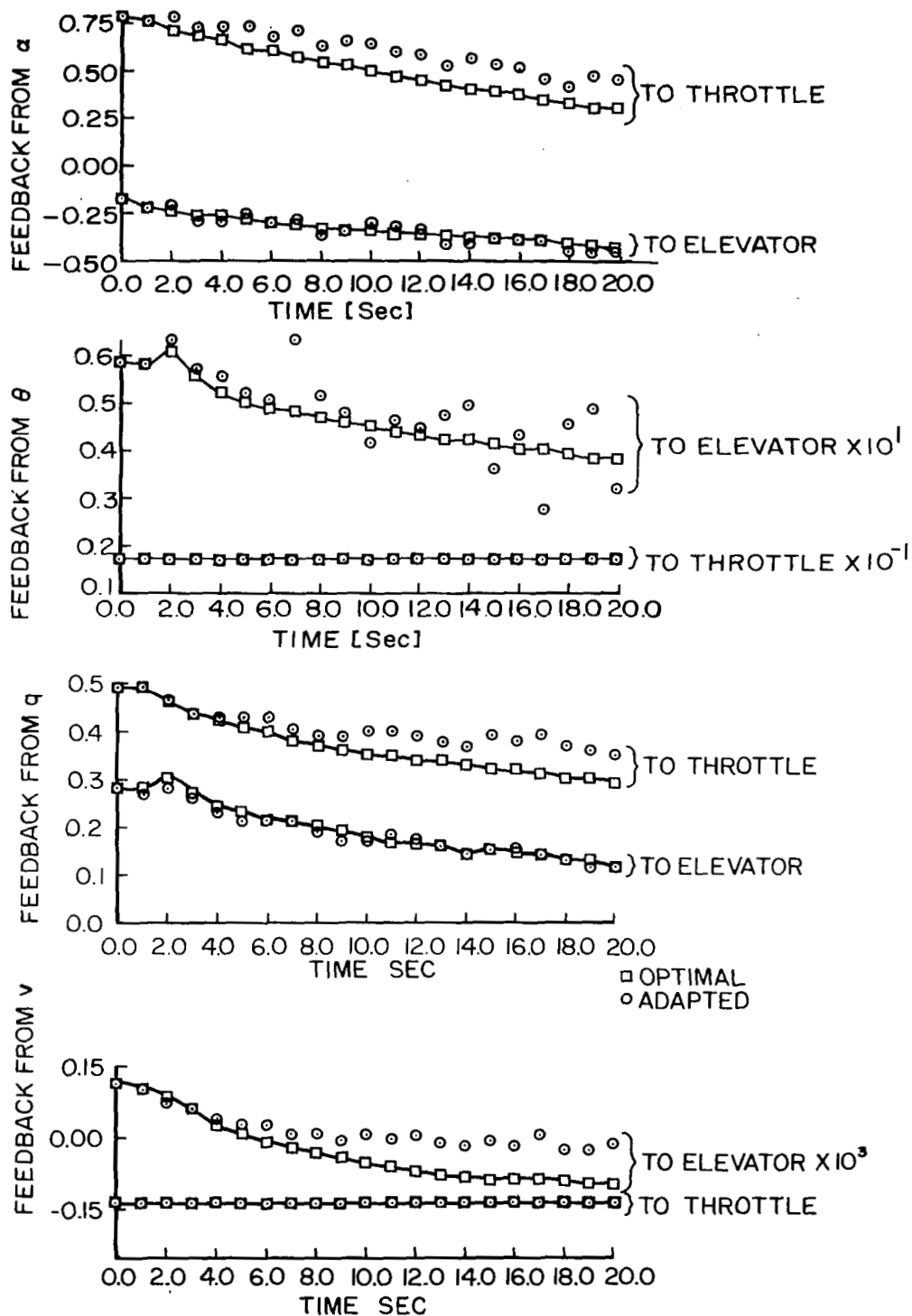


FIG. 4.11 FEEDBACK CONTROL GAIN ADAPTATION LONGITUDINAL
ADAPTIVE OPTIMAL REGULATOR, PILOT COMMAND =
0.1 rad, 0.2 hz ELEVATOR (Section 4.1.1)

well, whereas A_{23} and A_{43} were observed to have erratic responses. However, the model following being acceptable indicates that these two parameters were not influential to elevator excitation.

4.1.2 Single Stage Adaptive Controller Results

Several simulation experiments were performed using the single stage adaptive control logic with the lateral equations of motion in order to:

- . Compare the performance with that of the adaptive optimal regulator logic.
- . Evaluate the feasibility of not adapting the feedback gain K_{x_p} (eq. 3.24).
- . Assess the different identification algorithms presented in Section 3.2.
- . Assess performance in the presence of highly erroneous parameter estimates.

Some of the more salient of these procedures follow:

Experiment I

Objective:

To compare using lateral motion, the behavior of the stabilized single stage adaptive control system with the adaptive linear optimal regulator, when the number of identified parameters for each controller is 8 (rather than 12).

Procedure:

Both parameter and state estimates were obtained at each control sample period using the weighted least squares procedure discussed in Section 3.2.2. Feedback gains for the single stage controller were adapted using the Riccati update procedure described in Section 3.1.1.1.2.

Design Factors:

Pilot input: $\pm 5^\circ$ Aileron, 0.2 Hz square wave

Control sample period: 0.1 sec.

Gain update period: 1.0 sec.

Parameters identified: $A_{11}, A_{12}, A_{31}, A_{34},$
 $B_{11}, B_{12}, B_{22}, B_{31}$

Results and Discussion:

Fig. 4.12 depicts the roll rate responses for the single stage and the optimal regulator adaptive control logic. Corresponding roll rate feedback gains are shown in Fig. 4.13.

Observation of these curves (and other associated data) indicates that while both adaptive control algorithms are equally capable of generating the desired behavior, the single stage gains were slightly larger than those of the optimal regulator.

Experiment II

Objective:

To investigate the feasibility of not adopting the feedback gain in the single stage controller.

Procedure:

As in experiment I, weighted least squares procedures were used for both state and parameter estimation. Using the trajectory defined in Appendix C, and the lateral equations of motion, three cases were considered, namely:

- 1) Adapting the feedback gain K according to the procedures discussed in Section 3.1.1.2.
- 2) Fixing the feedback gain K so that it is stabilizing for all flight conditions. This was done using eqs. 3.23, 3.24 for the A of flight condition 3 and the B for flight condition 2, and the weights Q_1, R_1 from Section 3.1.3.4.
- 3) Fixing K in accordance with the procedures described by eqs. 3.25-3.31.

Results and Discussion:

Figure 4.14 which depicts the roll rate response for cases 1) and 2) shows that it is possible to select a feedback gain that eliminates the need for adaptation. Further tests which considered the effects of an unstable initialization in fact revealed better initial performance when the fixed feedback gain was used. Additional tests using no feedback gain at all (which is stabilizing since the given loop lateral dynamics are inherently stable) were, as expected, unsatisfactory.

With regard to the determination of a fixed feedback gain using the guaranteed cost procedures defined in eqs. 3.25-3.31, it was found that in order to satisfy these conditions, the feedback gains would be excessively large (on the order of 10^3).

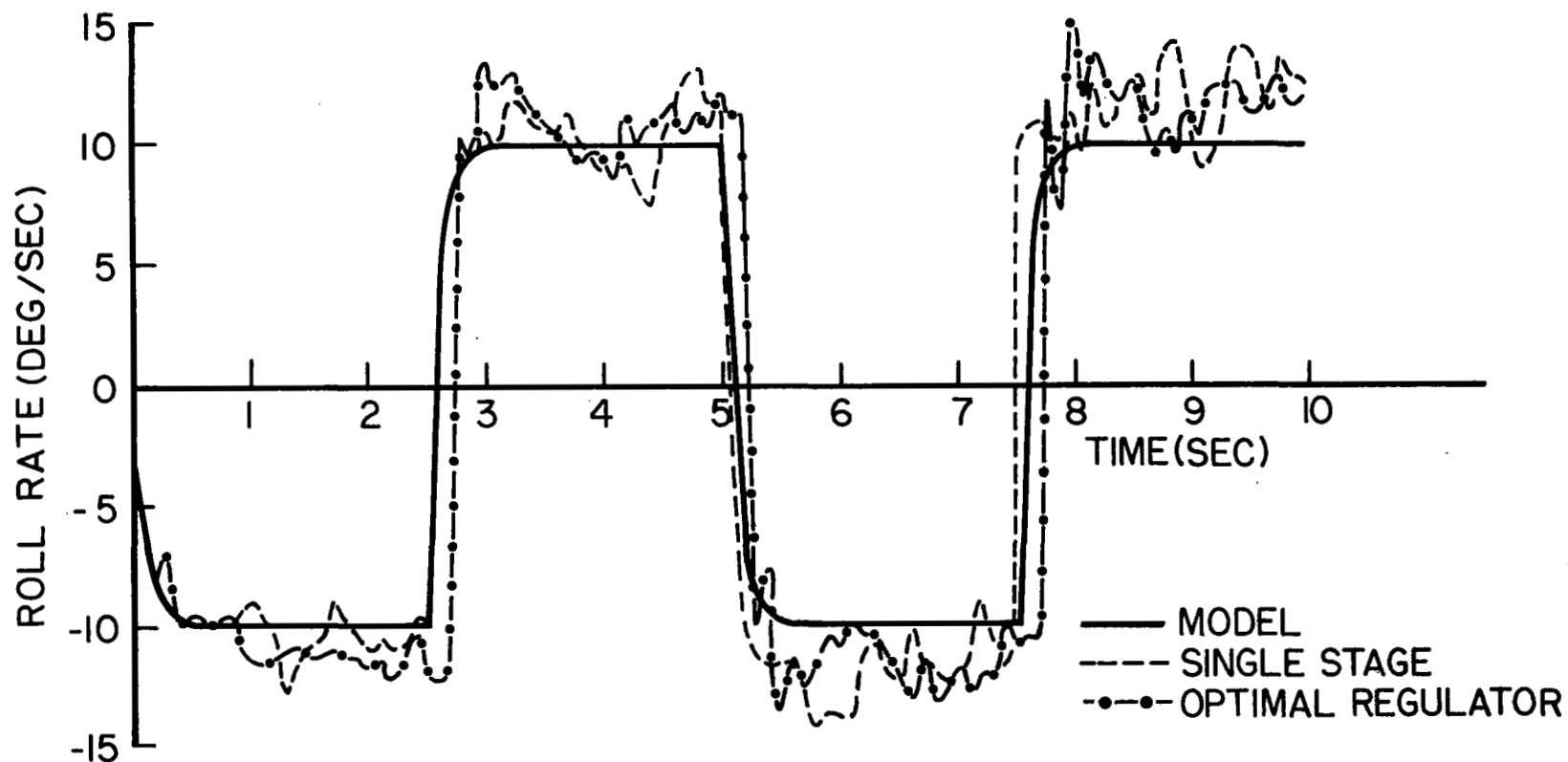
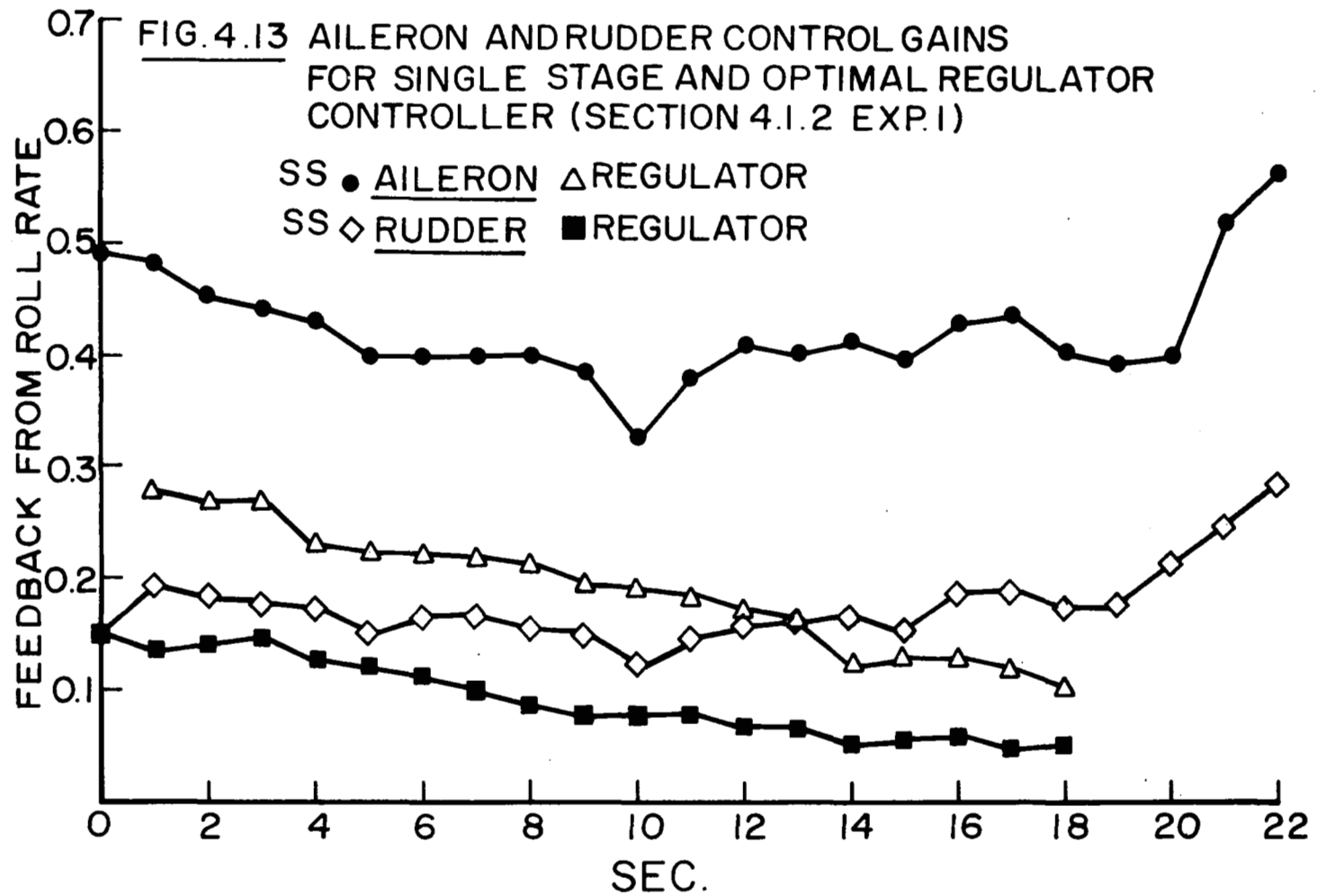


FIG. 4.12 ROLL RATE RESPONSE FOR OPTIMAL REGULATOR AND STABILIZED SINGLE STAGE ADAPTION LOGIC. 8 PARAMETERS IDENTIFIED (SECTION 4.12, EXP. I)



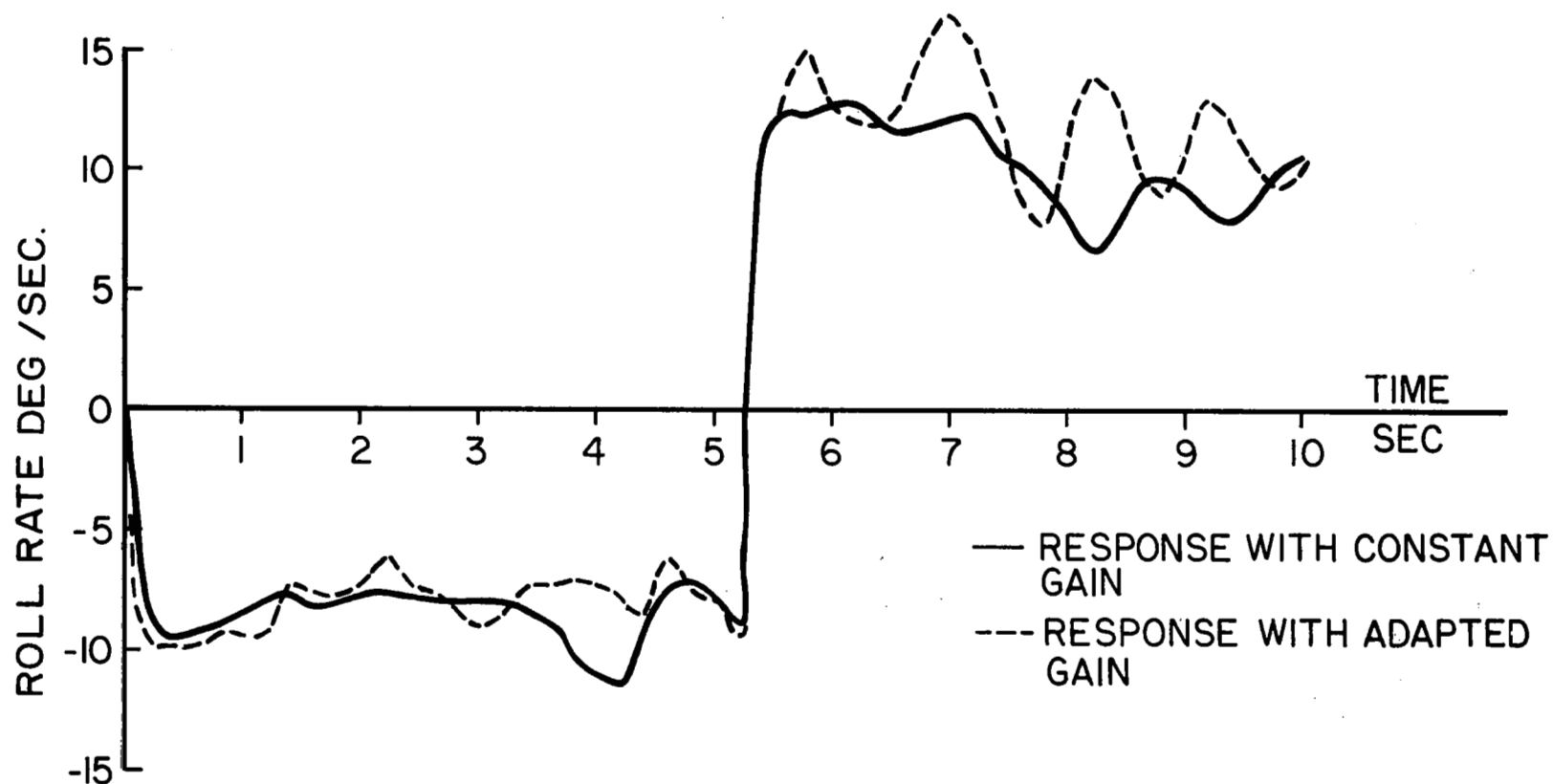


FIG. 4.14 ROLL RATE RESPONSE STABILIZED SINGLE STAGE CONTROLLER WITH AND WITHOUT ADAPTATION OF THE FEEDBACK GAIN (SECTION 4.1.2, EXP. II)

Experiment III

Objective:

To assess the performance of different state and parameter estimation algorithms when parameter estimates are initially 50% of the corresponding true values.

Procedure:

Using the noise sequences defined in Appendix B, estimation was performed with the weighted least squares, minimum variance and extended Kalman filter procedures. Control was achieved using the stabilized single stage algorithm with all gains being adjusted. The aircraft was simulated to be fixed at flight condition 2. Those parameters being identified were initialized at 50% of their true values.

Constant Factors:

Gain adaptation period: 0.2 sec.

Control sample period: 0.2 sec.

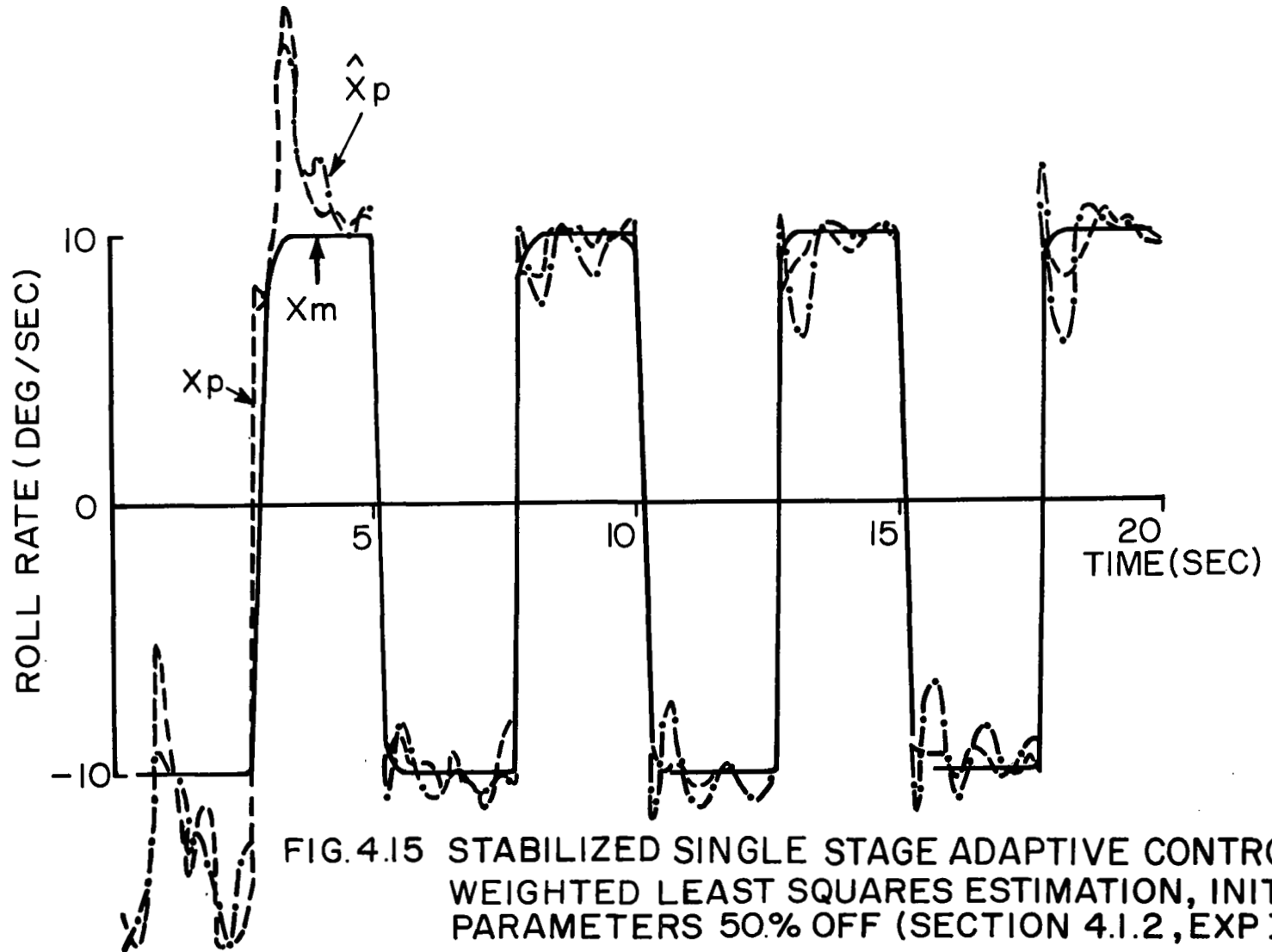
Parameters identified: 1st and 3rd rows of A_p , B_p

Pilot input: $\pm 5^\circ$ aileron, 0.2 hz square wave

Results and Disucssion:

Roll rate responses achieved using the weighted least squares algorithm and the extended Kalman filter are shown in Figures 4.15 and 4.16 respectively. These results indicate that although both algorithms were capable of recovering from the erroneous initial conditions, the weighted least squares procedure was much more responsive. This, in fact, was reflected not only in the depicted responses but also in the adapted values for the gains.

With regard to the effects of using the minimum variance algorithm, it was observed that the roll rate response was initially very oscillatory, and that the parameter estimates and corresponding adapted gain values were closer to their true values than those computed using the extended Kalman filter and the weighted least squares procedure. The initial oscillation in roll rate can be explained by noting from eq. 3.71 that the initial uncertainty in the state estimates, P_x is directly related to the parameter uncertainty P . Thus an initial high uncertainty in the parameter can result in state estimates that tend to track the measurement noise. Because the control signal itself includes products of the adapted gains with the state estimates, the poor initial state estimates tend to degrade the overall response.



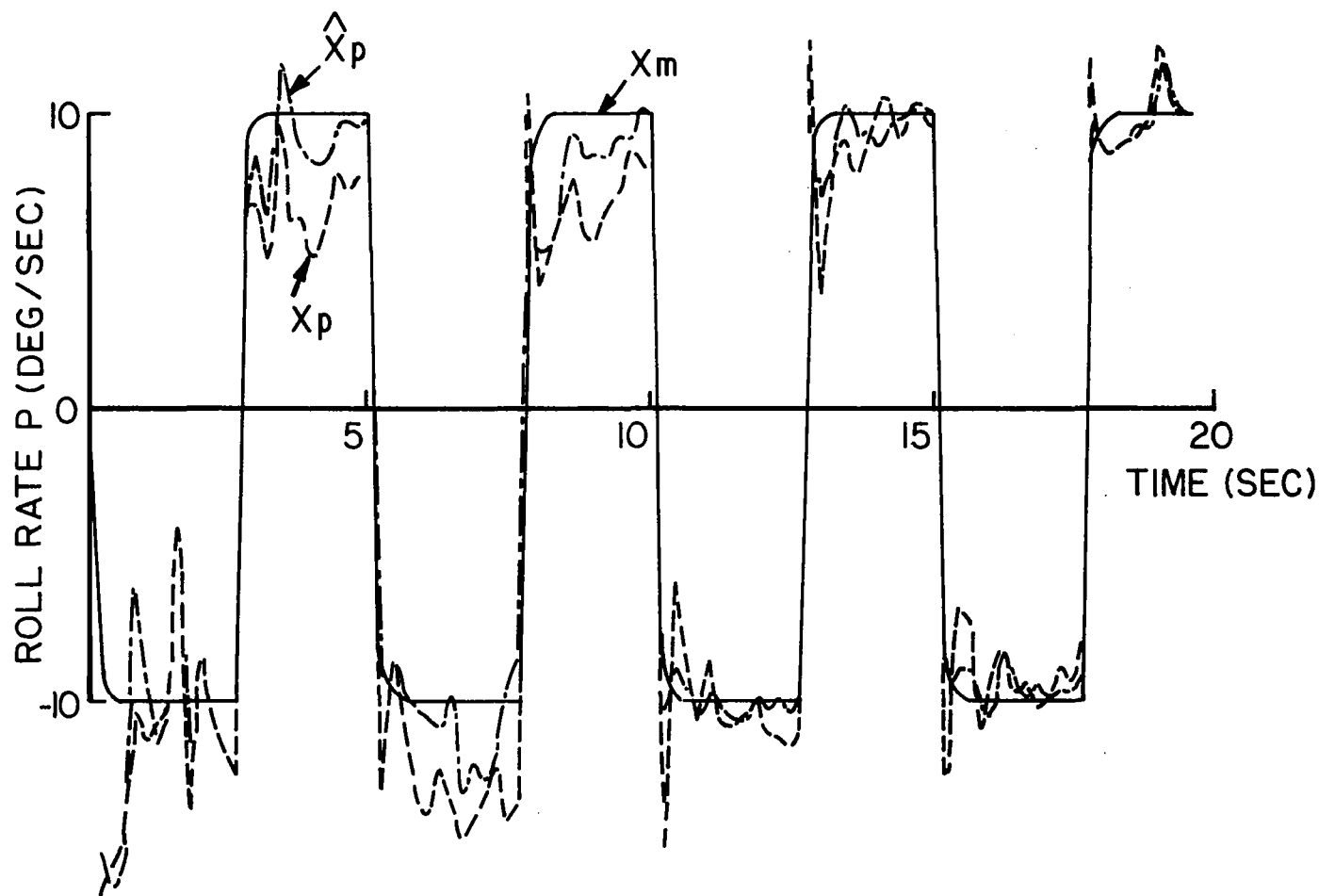


FIG. 4.16 STABILIZED SINGLE STAGE ADAPTIVE CONTROLLER
EXTENDED KALMAN ESTIMATION, INITIAL PARAMETERS
50% OFF, (SECTION 4.1.2, EXP. III)

Experiment IV

Objective:

To study the capability for recovering from a set of erroneous parameter estimates which yield a set of destabilizing gains.

Procedure:

Weighted least squares procedures were used for both state and parameter estimation in the presence of measurement noise sequences as defined in Appendix B. All parameter estimates were initialized at the average values as computed over the six given flight conditions, when in reality the aircraft was initialized at flight condition 2. The corresponding initial gains were such that the closed loop system was initially unstable.

Constant Factors:

Gain adaption period = 0.2 sec.

Constant sample period = 0.2 sec.

Parameters identified: 1st and 3rd rows of A_p and B_p .

Pilot input: $\pm 5^\circ$ aileron, 0.1 hz square wave

Results and Discussion:

Figures 4.17, 4.18, 4.19 respectively depict the roll rate response, the estimates for $B(1,1)$, and the roll rate and sideslip feedback gains for the situation in which the aircraft remains at flight condition 2. Similar responses (except for the gains) are given in Figures 4.20, 4.21 for the case in which the aircraft continues along the trajectory defined in Appendix C.

The results show that even with the use of initially unstable gains, the system is able to adapt to a stable operation. This is due to the excellent response of the identifier to a very large and rapidly varying signal. This, as shown in Figure 4.19, leads to a set of control gains which are quickly adapted towards their optimal values.

4.1.3 Implicit Adaptive Controller Results

The feasibility of using an implicit adaptive flight controller was examined by applying the controller of eq. 3.32 to the linearized lateral equations of motion. To this effect Figures 4.22 and 4.23 show the potential of this method for improving the model following for the following conditions:

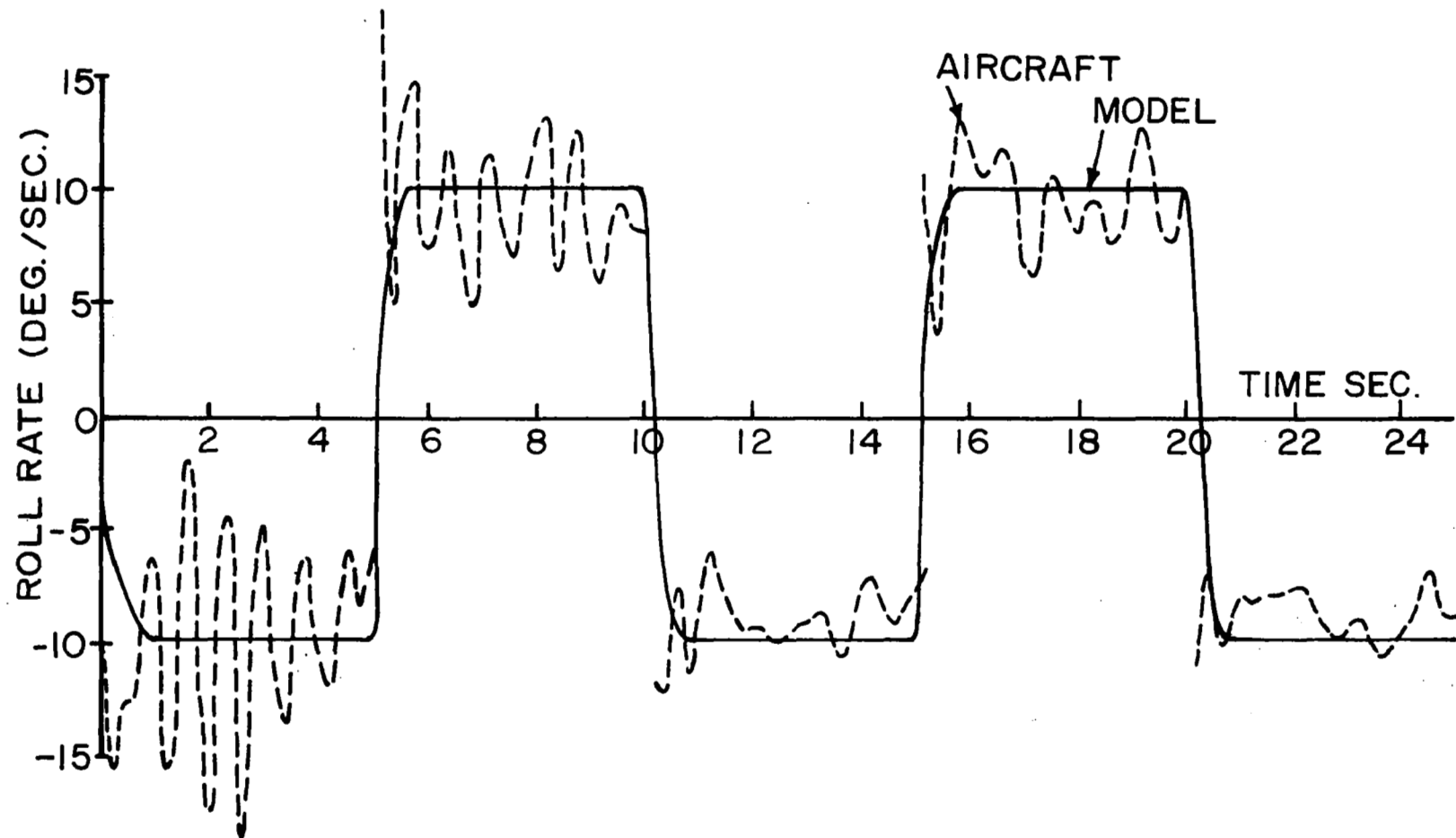


FIG.4.17 STABILIZED SINGLE STAGE CONTROLLER,FLIGHT CONDITION 2
INITIAL PARAMETERS AT AVERAGE VALUE,(UNSTABLE INITIALIZATION)
SECTION 4.1.2, EXP IV.

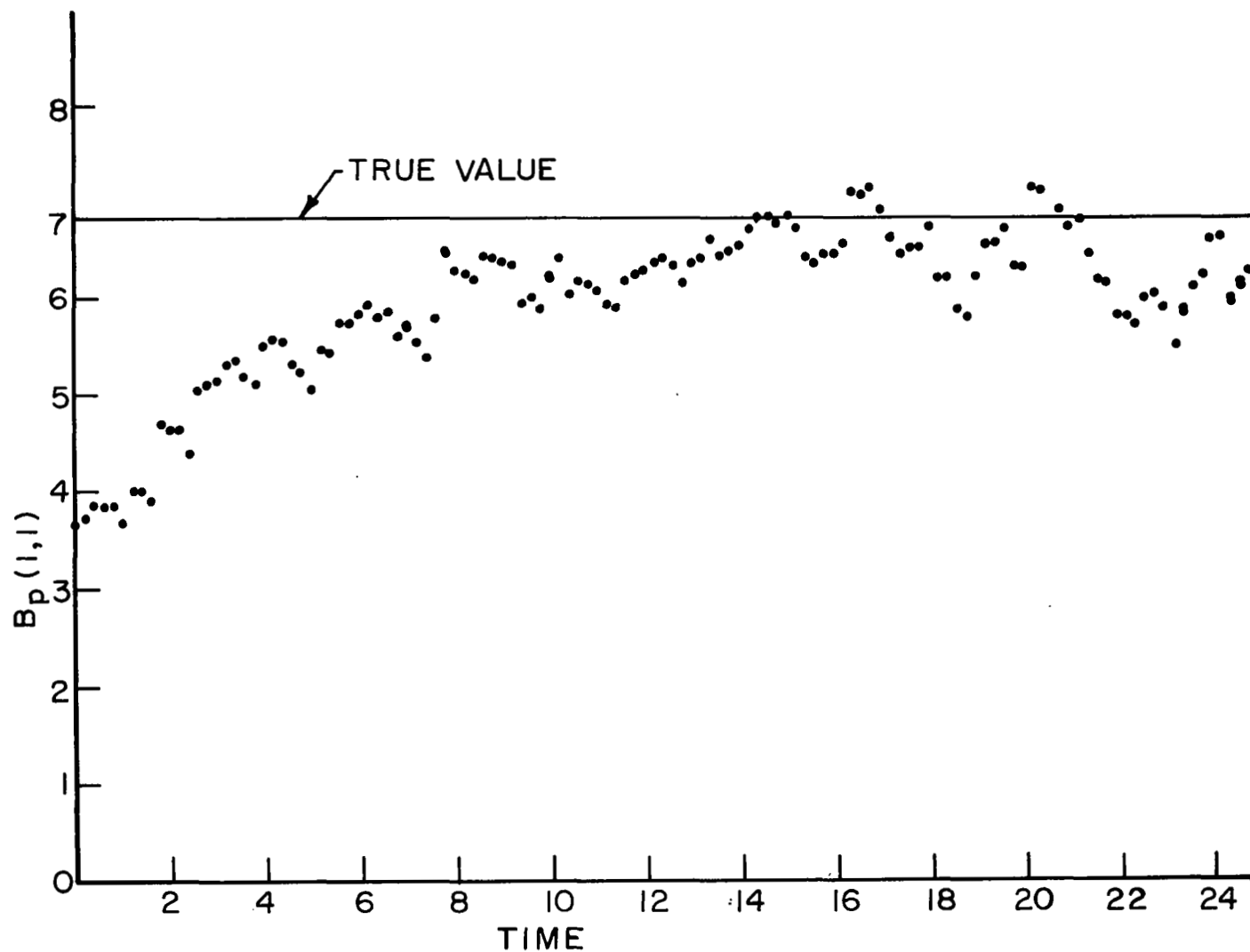


FIG. 4.18 $B_p(1,1)$ ESTIMATES, FLIGHT CONDITION 2, INITIAL PARAMETERS AT AVERAGE VALUE, (UNSTABLE INITIALIZATION), SECTION 4.1.2, EXP. IV.

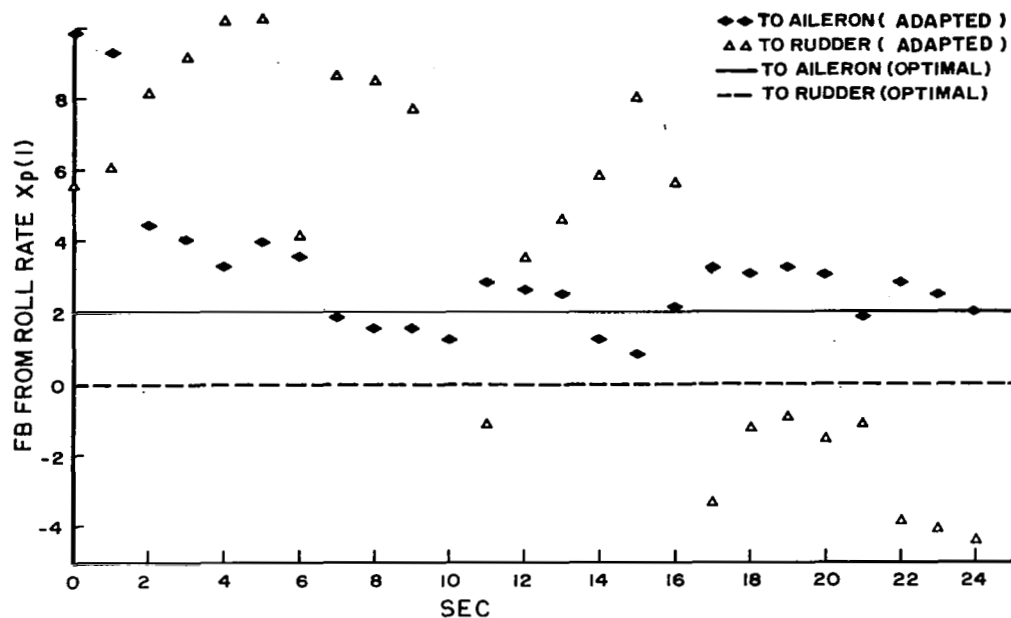


FIG. 4.19A ROLL RATE FEEDBACK GAINS, STABILIZED SINGLE STAGE CONTROLLER, FLIGHT CONDITION 2, INITIAL PARAMETERS AT AVERAGE VALUES (UNSTABLE INITIALIZATION) SECTION 4.1.2, EXP. IV

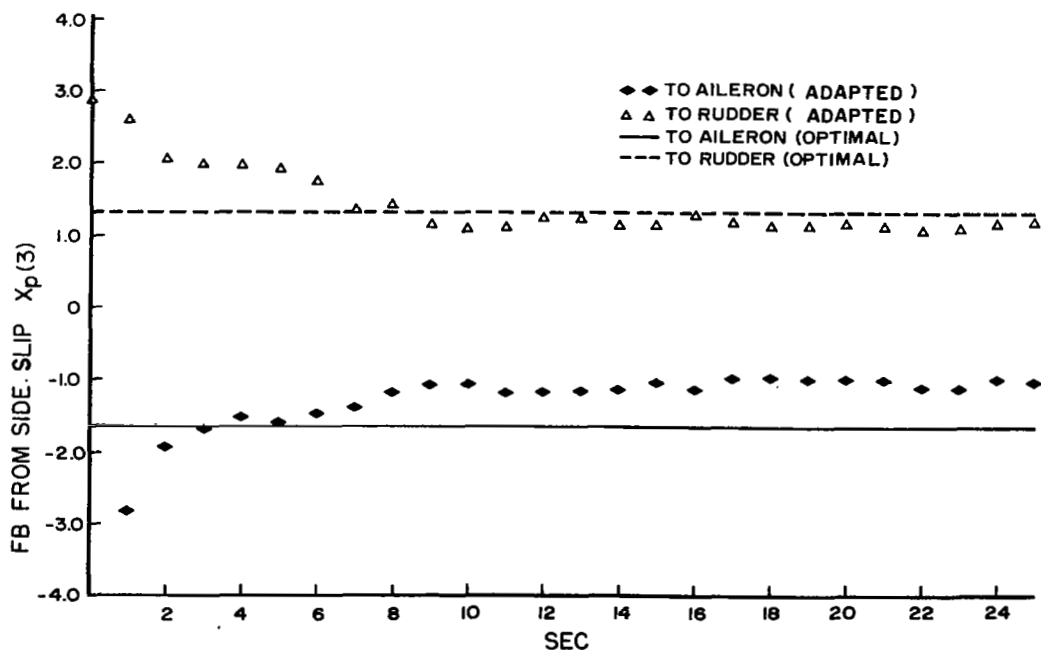


FIG. 4.19B SIDESLIP FEEDBACK GAINS, STABILIZED SINGLE STAGE CONTROLLER, FLIGHT CONDITION 2, INITIAL PARAMETERS AT AVERAGE VALUES (UNSTABLE INITIALIZATION) SECTION 4.1.2, EXP. IV

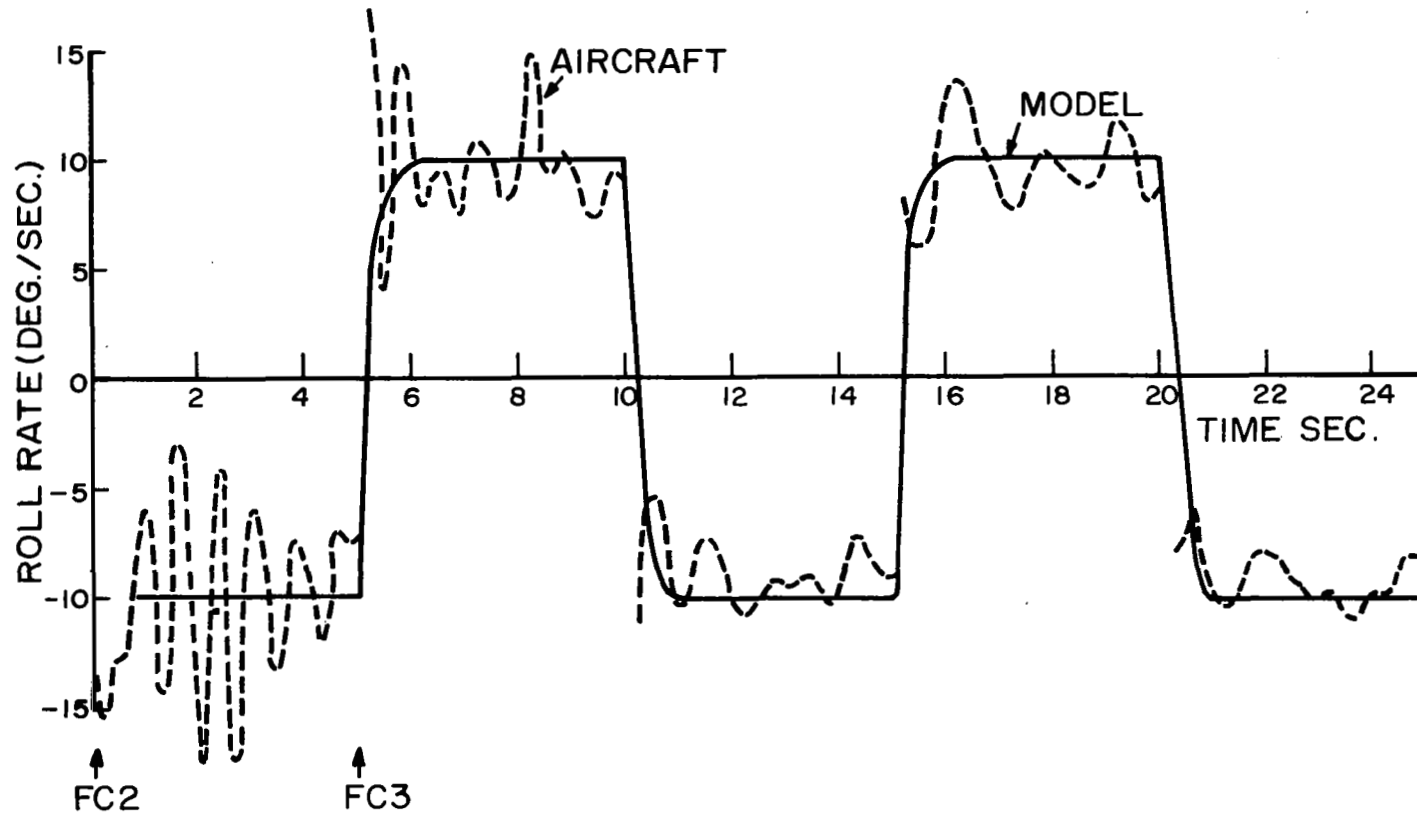


FIG. 4.20 STABILIZED SINGLE STAGE CONTROLLER, APPENDIX C TRAJECTORY STARTING AT FC 2, INITIAL PARAMETERS AT AVERAGE VALUE (UNSTABLE INITIALIZATION) SECTION 4.2.1, EXP IV

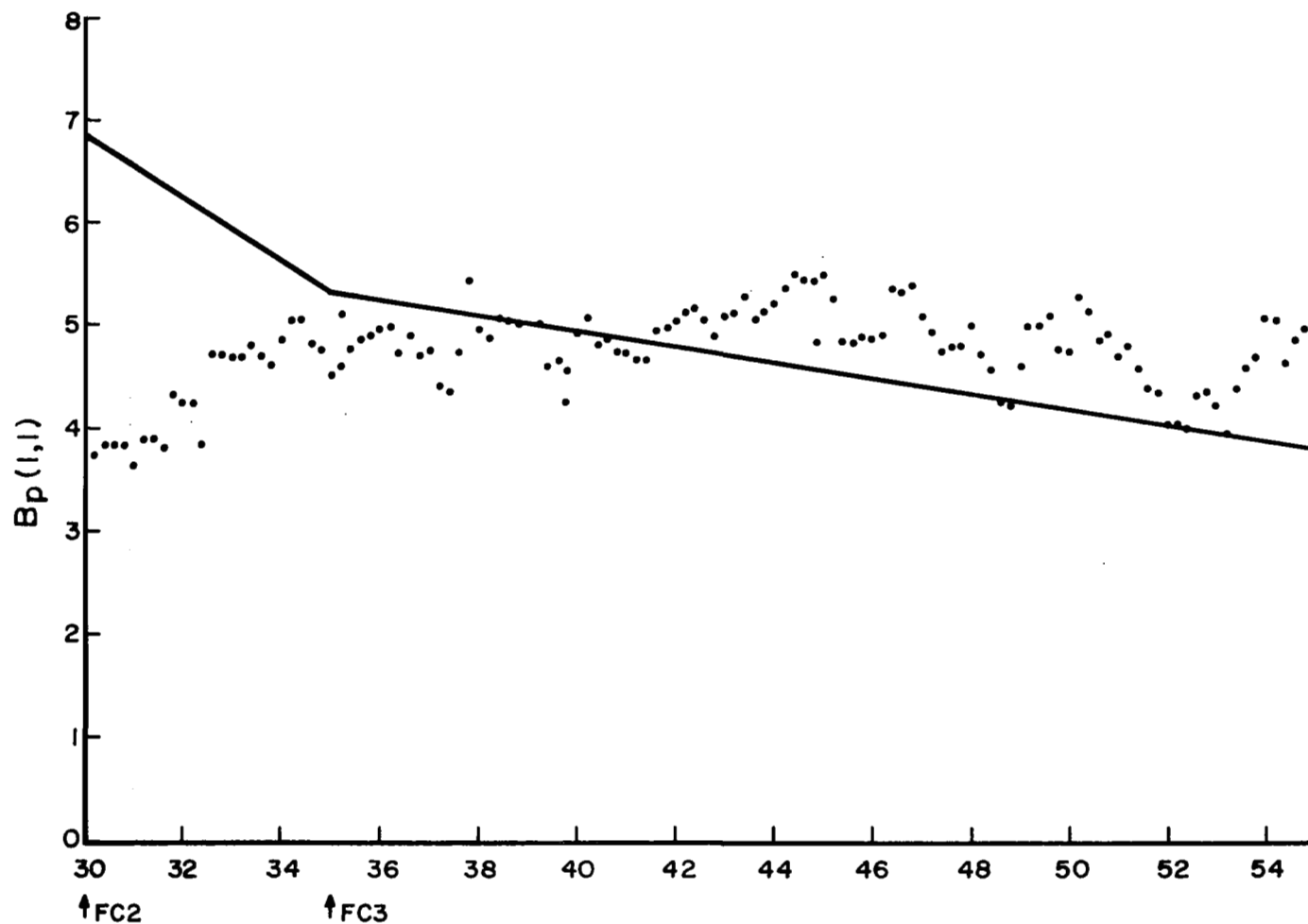


FIG. 4.2I $B_p(1,1)$ ESTIMATES, STABILIZED SINGLE STAGE CONTROLLER, APPENDIX C TRAJECTORY STARTING AT FC2, INITIAL PARAMETERS AT AVERAGE VALUE (UNSTABLE INITIALIZATION) SECTION 4.2.1, EXP IV

Flight condition 1

$$\text{Initial gains: } K_{x_p} = 1.5 [K_{x_m} - B_p^+ (A_m - A_p)]$$

$$K_{u_m} = B_p^+ B_m$$

K_{x_m} chosen to stabilize (A_m, B_p)

$$D = \begin{pmatrix} .618 & .0718 & -.147 & 6.88 \\ .456 & -2.40 & 8.24 & -.0749 \end{pmatrix}$$

$$Q = \begin{pmatrix} 10^4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 10^4 & 0 \\ 0 & 0 & 0 & 10^4 \end{pmatrix} \quad R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Sampling time = .05 sec.

Of interest is the capability to eliminate the large steady state error in roll rate which results from the poor initial gain selection. (Fig. 4.22) This reduction in steady state error is a result of the significant adaptation of the feedforward gain matrix K_{u_m} (see Fig. 4.23). Adjustments of the feedback gain K_{x_p} were noted however to only be about 10%.

Preliminary testing using the augmented controller (eq. 3.40) resulted in a stable but not satisfactory response. It is anticipated that this can in the future be remedied by further tuning of the appropriate weighting matrices.

4.2 Nonlinear System Evaluation

In order to assess the applicability of the linearized or perturbation adaptive controller to an actual aircraft, tests were made using NASA Langley's batch simulation of the nonlinear six degree-of-freedom equations of motion. Initially for purposes of testing the feasibility of just the perturbation control algorithms, small magnitude symmetrical square wave pilot commands were applied in order to maintain a static flight condition. These tests showed that both the optimal linear regulator controller and the single stage controller were effective in controlling the incremental states of the simulated aircraft.

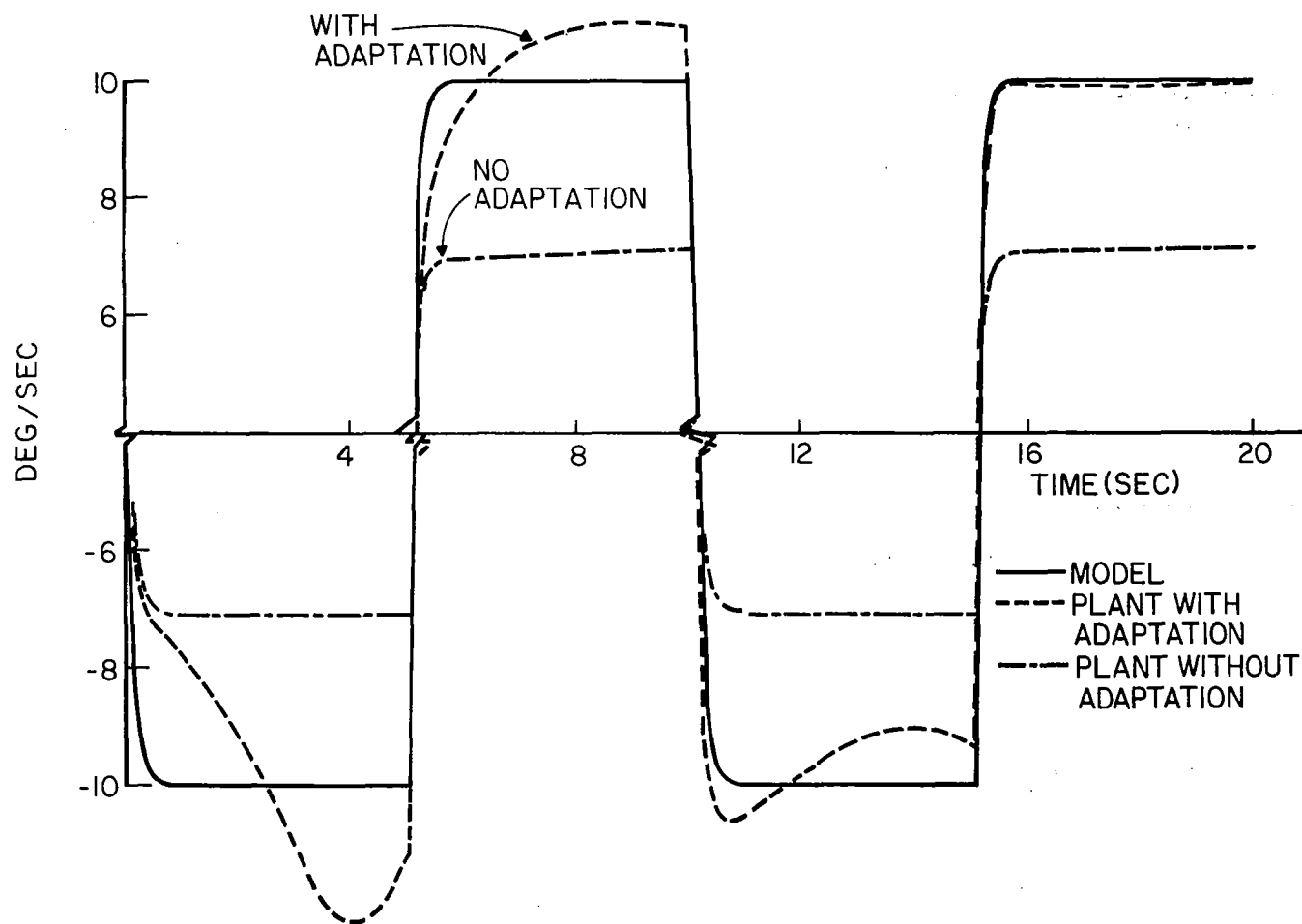


FIG. 4.22 IMPLICIT ADAPTION ROLL RATE VS. TIME (SECTION 4.1.3)

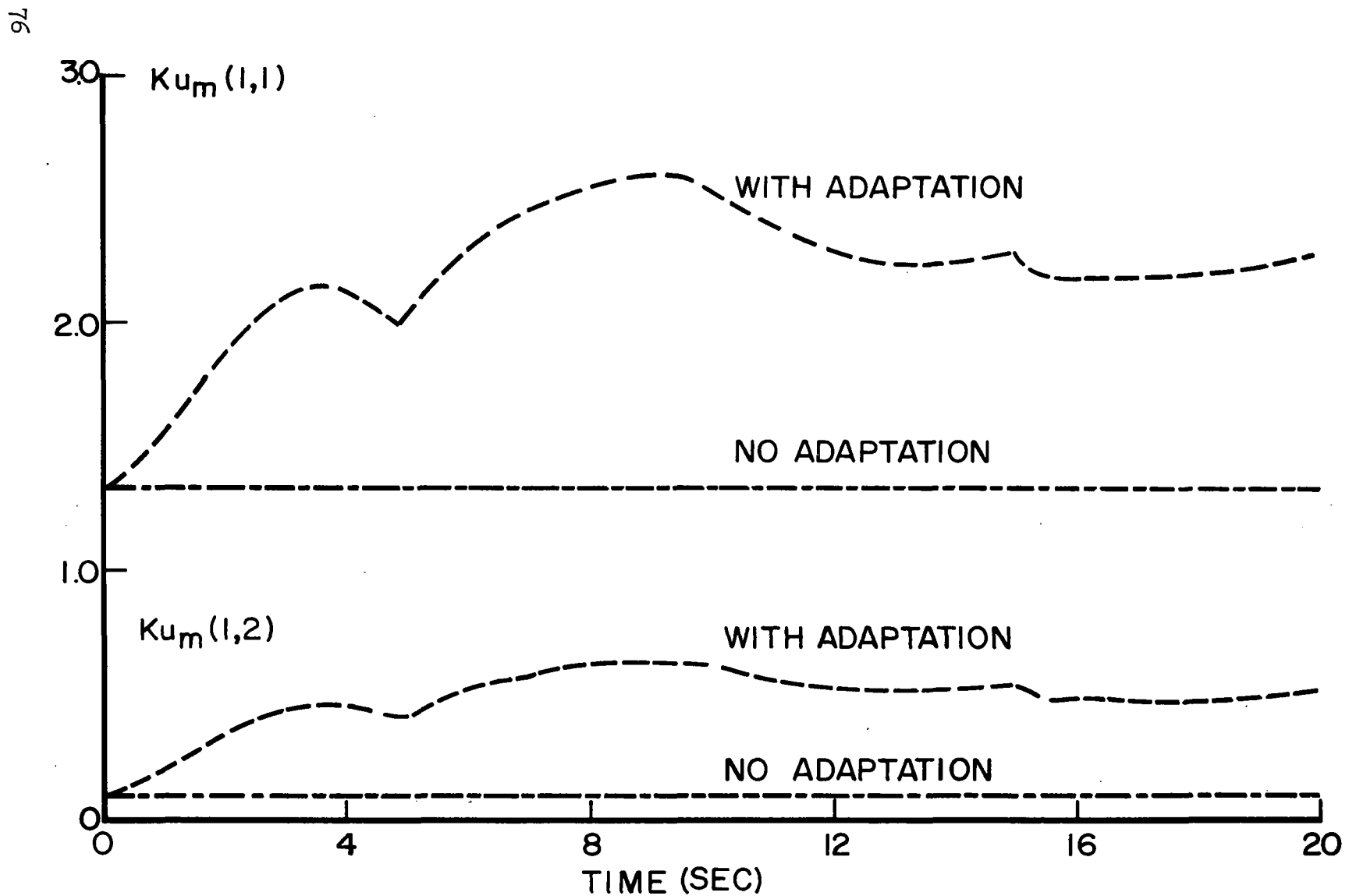


FIG.4.23 IMPLICIT ADAPTATION FEED FORWARD GAIN ADJUSTMENTS
(SECTION 4.1.3)

Consequently further experiments were made in order to evaluate the overall adaptive system including the identifier and trim update filters. Procedures considered for trim computation included:

- . Holding all trim values constant. This is valid however only for small perturbations about a given flight condition.
- . Computing through the use of washout filters the trim values for $v, \alpha, \theta, \delta_e, \delta_T$ and assuming zero trim values for $p, r, \beta, \phi, q, \delta_a, \delta_r$.
- . Computing through the use of washout filters, the trim values for all states and controls.

These washout filters for the states were defined by the low pass filter equation:

$$\dot{x}_i^t + \frac{1}{\tau} x_i^t = \frac{1}{\tau_i} x_i^a$$

where:

$$x_i^t = \text{trim state}$$

$$x_i^a = \text{total state}$$

The filter time constants τ_i were set equal to three times the largest time constant τ_m for the corresponding model states as tabulated below.

<u>x</u>	<u>τ_m (sec)</u>	<u>$\tau_i (= 3 \max \tau_m)$</u>
q	.91	2.73
v	10.00	30.00
α	.91	2.73
θ	.91	2.73
p	1.40	4.20
r	1.40	4.20
β	1.40	4.20
ϕ	1.40	4.20

With respect to forming trim values for the control signals, various possibilities must be considered in view of the presence of both the pilot command \underline{u}_m and the actual applied control \underline{u}_p . These include:

- . Computing a trim value for \underline{u}_p using a washout filter, and letting \underline{u}_m denote the actual incremental command.
- . Forming incremental values of \underline{u}_p as above and forming incremental values of \underline{u}_m by subtracting out from the absolute stick motion the trim values for \underline{u}_p . These in turn can be formed by either washing out \underline{u}_m or by using the trim values of \underline{u}_p . It should be noted that this procedure applied to step changes in \underline{u}_m will result in an incremental command that varies from the initial step change toward zero in accordance with the time constant of the washout filters. This is reasonable if after a sufficiently long time, the new stick position is to be regarded as a new trim position.

In all cases it is recommended that either periodically or upon detection of a step change in stick position that the model incremental state vector be reset equal to the aircraft's incremental state vector. This is in keeping with the model following philosophy.

To thus evaluate the applicability of the linearized digital adaptive controller, the optimal adaptive regulator logic was incorporated into Langley's nonlinear simulation and tested according to the following outline.

Objective:

To evaluate the response of the overall adaptive system with and without washout filters and to assess the effectiveness of the linearized adaptive control algorithms.

Procedure:

The time constants for the filters on the controls were selected as follows:

<u>Control</u>	<u>τ_i</u>	
δ_e	2.73	same as q
δ_T	30.00	same as V
δ_a	1.40	same as p
δ_r	1.40	same as p

The states and trim were initialized to values corresponding to an altitude of 610m (2000 ft.) and a Mach number equal to 0.3. The parameters to be identified were initialized at 50% of their true values while all other parameters were held constant at their true initial values.

Design Factors:

Pilot commands:

+ .01 r, $0 \leq t \leq 5$ sec

Aileron deflection

0 r, $5 \text{ sec} \leq t \leq 10 \text{ sec}$

- .01 r, $10 \text{ sec} < t \leq 15 \text{ sec}$

0 r, $15 \text{ sec} < t \leq 20 \text{ sec}$

or elevator deflection of 0.3° step

Control sample time = 0.1 sec

Identification time = 0.1 sec

Gain update time = 1.0 sec

The following four cases were tested:

- (1) Constant Trim, No Adaptation
- (2) Constant Trim, Fully Adaptive
- (3) Washout trim correction of all aircraft states and controls, pilot command \underline{u}_m corrected for trim variation.
- (4) Washout trim correction of $v, \alpha, \theta, \delta_e, \delta_r$.
No change of pilot command \underline{u}_m

Results and Discussion:

Figures 4.24a, b, c, d depict the behavior of δ_a, δ_r, p , and β in response to the aileron command for the non-adaptive online regulator logic (case 1). The significant improvement resulting from adaptation with a constant trim value is evident in Figures 4.25a, b, c, d (i.e. case 2). Of importance is the capability of the adaptive controller for removing the transient oscillation in roll rate and for significantly reducing the magnitude of the sideslip angle. It should be noted that the excessive initial oscillations exhibited under adaptive control can be attributed to the transient response of the identifier. These, however, did dissipate within three seconds.

It was further observed that correcting \underline{u} for changes in trim computed by washing out either \underline{u}_m or \underline{u}_p (case 3) resulted in a tendency for the model to be too sluggish. Thus it was decided in further tests to let \underline{u}_m denote the incremental pilot command without any trim correction. This is realistic assuming that incremental stick motion can be sensed by the fly-by-wire logic. Case 4 results for an elevator step command to the modified single stage adaptive logic are shown in Figures 4.26a, b, c.

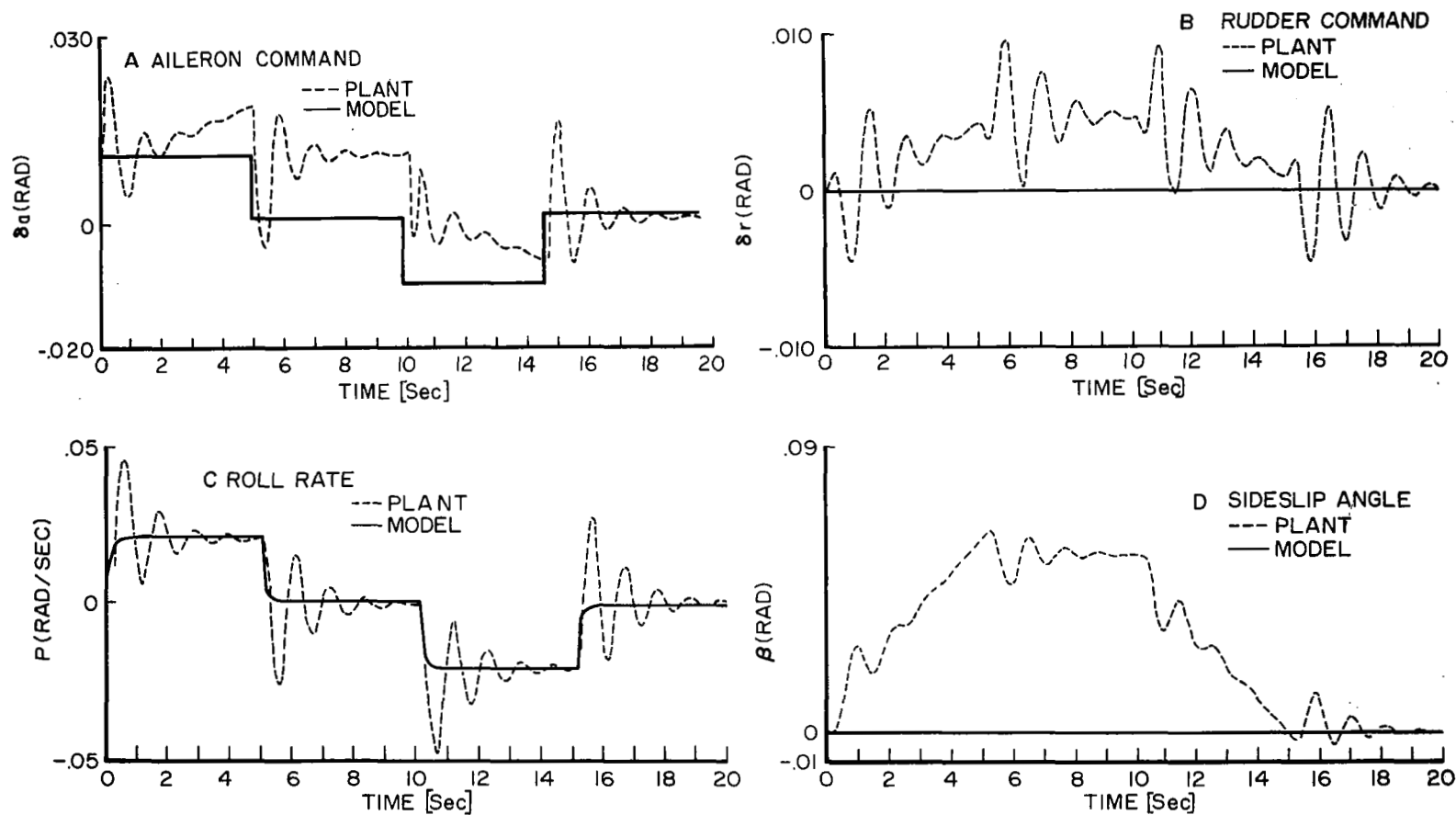


FIG. 4.24 EVALUATION OF THE OPTIMAL REGULATOR CONTROLLER USING THE NONLINEAR EQUATIONS OF MOTION (SECTION 4.2) NO ADAPTION

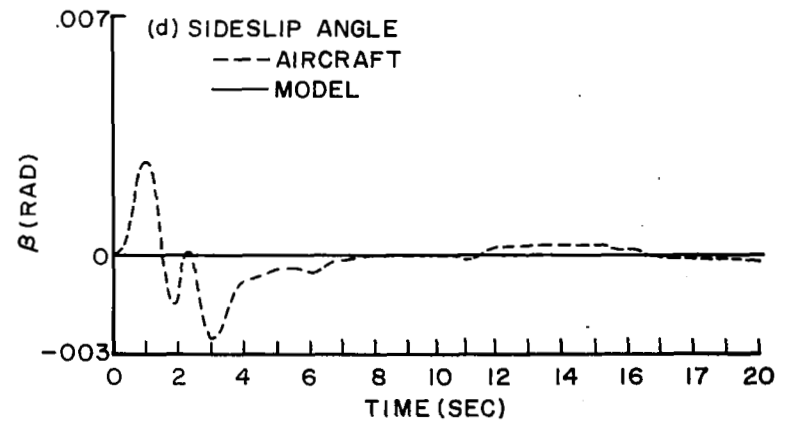
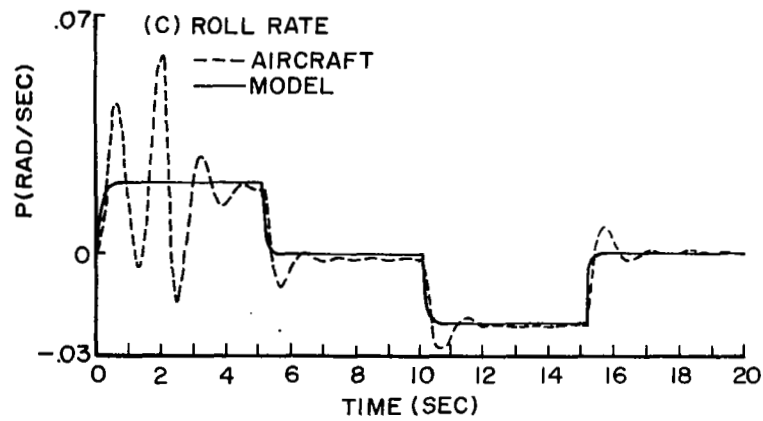
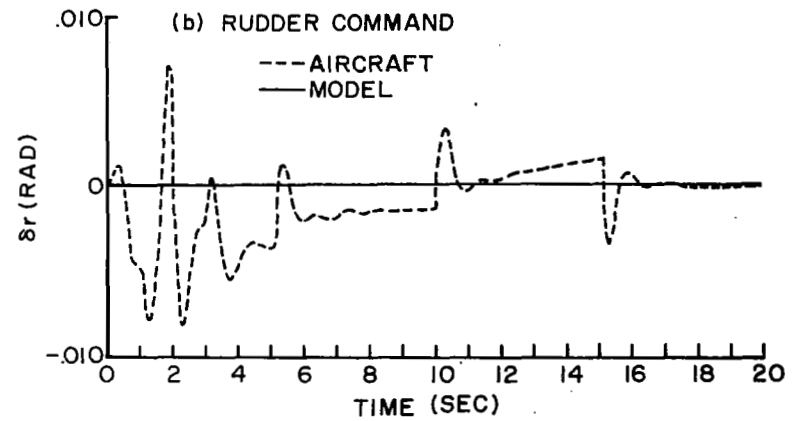
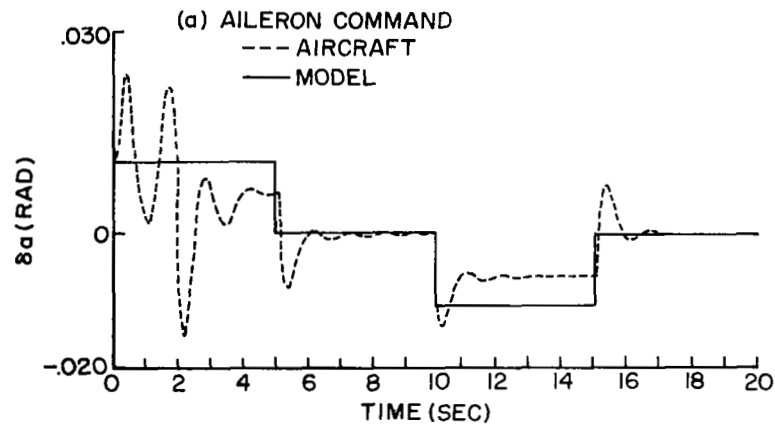


FIG. 4.25 EVALUATION OF THE ADAPTIVE OPTIMAL REGULATOR
CONTROLLER USING THE NONLINEAR EQUATIONS OF MOTION
(SECTION 4.2)

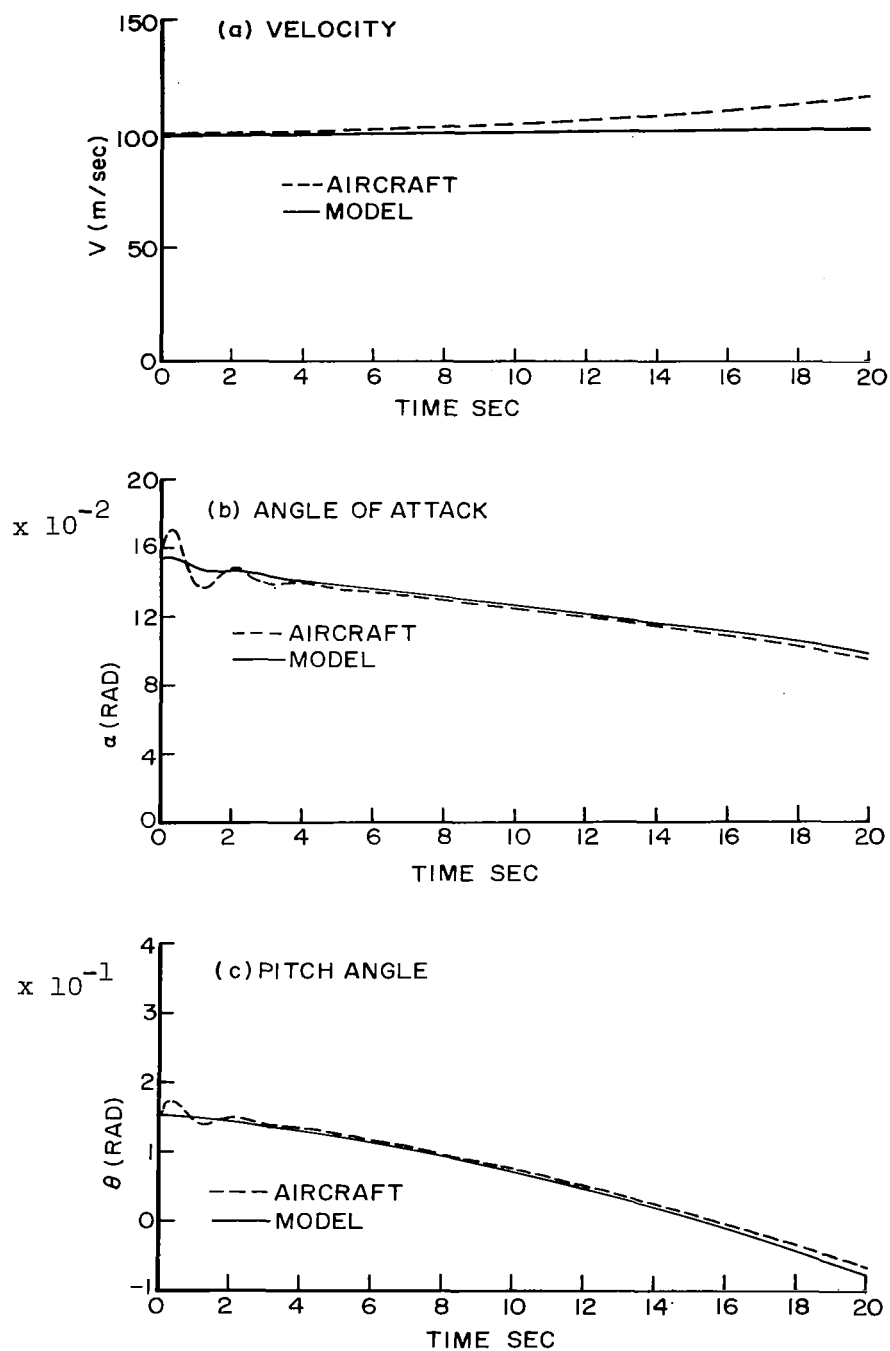


FIG. 4.26 EVALUATION OF THE ADAPTIVE SINGLE STAGE CONTROLLER USING THE NONLINEAR EQUATIONS OF MOTION. PILOT COMMAND = 3° ELEVATOR STEP. (SECTION 4.2)

As a guide towards determining the timing specifications for the various adaptive control functions, the CDC-6600 nonlinear simulation operating in batch mode required the following time slices:

- . Recursive identification of 16 parameters 64 ms
- . Adaptation of the optimal regulator gains for
both lateral and longitudinal motion 102 ms
(using eq. 3.11)
- . Computation of δ_a , δ_r , δ_e , and δ_T 28 ms

Since it is not necessary to perform adaptation every sample period, these results indicate that the proposed digital adaptive controller can be operated without problems at least 10 times per second.

More prudent programming procedures and the use of machine language coding might in fact double or triple this allowable frequency.

5. CONCLUSIONS AND RECOMMENDATIONS

Based upon both the analytical and simulation efforts described in the previous chapters, the following recommendations relative to implementation can be made:

- . The two explicit adaptive controllers designed using stabilized single stage logic and optimal regulator logic are feasible for on-board application. This conclusion is based upon analysis and simulation efforts using both the linear and nonlinear equations of motion.
- . Online estimation of the states and parameters is best performed by a procedure which first utilizes the noisy measurements directly for parameter estimation, and then utilizes these parameter estimates with the state measurements for state estimation.
- . The explicit adaptive controllers are capable of rapid recovery from highly erroneous parameter estimates which could in fact define a set of destabilizing gains. This follows because the relatively large oscillations in the aircraft states will result in the identifier's being forced by large signal to noise signals with a large degree of fluctuation. Thus, rapid convergence towards the proper parameter values will take place.
- . On-board implementation of the proposed linearized designs is achievable if washout filters are used for computation of trim states and controls.

- . Implicit adaptive control logic should not as yet be implemented. Although the resulting system will be stable, no procedure has been found for tuning all the pertinent weighting factors so as to yield favorable response characteristics.

Recommendations for future efforts include the following:

- . Determine procedures for designing implicit adaptive controllers in accordance with desired system behavior specifications. Such procedures will, by eliminating the need for an online identifier, reduce the complexity of the adaptive system and further ease implementation.
- . Program the two explicit adaptive control algorithms into the actual flight computer and interface it with NASA Langley's simulation of the nonlinear equations of motion. This will enable a more effective evaluation of the storage, timing, and interface requirements of the controllers.
- . Determine the effects of directly incorporating the bending modes into the state equations rather than into the sensor noise characteristics. Recall from Section 2.1.3 that the correlated measurement noise sequences were selected so as to reflect both sensor noise and bending effects. Modelling the bending modes with additional state equations will permit the use of wider band noise sequences with smaller variances.

REFERENCES

1. Kass, P. J., "Fly by Wire Advantages Explored," Aviation Week and Space Technology, July 10, 1972, pp. 53-54.
2. Sutherland, Major J., "Fly by Wire Control Systems," AGARD Conference Proceedings No. 52, Advanced Control System Concepts, Sept. 1968, pp. 51-72.
3. McGough, J.; Moses K.; Platt, W.; Reynolds, G.; Strole, J.; "Digital Flight Control System Redundance Study", AFFDL - TR-74-83, Bendix Corp., July 1974.
4. Smyth, R. and Ehlers, H. L., "Survey of Adaptive Control Applications to Aerospace Vehicles," AGARD Conference Proceedings No. 52, Advanced Control System Concepts, Sept. 1968, pp. 3-13.
5. Ostgaard, M. A., "Case for Adaptive Control," AGARD Conference Proceedings No. 52, Advanced Control System Concepts, Sept. 1968, pp. 15-27.
6. Kaufman, H.; Alag, G.; Berry, P.; Kotob, S; "Digital Adaptive Flight Controller Development," NASA CR-2466, Dec. 1974.
7. "Advanced Concepts for Guidance and Control of Tactical Missiles," TR-170-5, The Analytical Sciences Corporation, June 30, 1973.
8. Winsor, C. A. and Roy, R., "Design of Model Reference Adaptive Control Systems by Liapunov's Second Method," IEEE Trans. on Automatic Control, April 1968, p. 204.
9. Landau, I.D. and Courtiol, B., "Adaptive Model Following Systems for Flight Control and Simulation," AIAA Paper No. 72-95, AIAA 10th Aerospace Sciences Meeting, San Diego, California, Jan. 1972.
10. Asseo, S. J., "Application of Optimal Control to Perfect Model Following," JACC, June 1968.
11. Picon-Jiminez, J. and Montgomery, R. C., "Design of a Digital Adaptive Control System for Reentry Vehicles," 9th Allerton Conference on Circuit and Systems Theory, Monticello, Ill., 1971.
12. Chan, Y. T., "Perfect Model Following with a Real Model," 1973 JACC, pp. 287-293.
13. Erzberger, H., "On the Use of Algebraic Methods in the Analysis and Design of Model-Following Control Systems," NASA TN D-4663, Ames Research Center, July 1968.

14. Alag, G., and Kaufman, H., "Digital Adaptive Flight Control Design Using Single Stage Model Following Indices," 13th Symposium on Adaptive Processes, Phoenix, Nov. 1974.
15. Motyka, P. R., "Variability Stability Simulation Techniques for Nonlinear Rate Dependent Systems," 1972 JACC, pp. 266-271.
16. Alag, G., and Kaufman, H., "Digital Adaptive Model Following Flight Control," 1974 AIAA Mechanics and Control of Flight Conf., paper No. 74-886, Anaheim, Aug. 1974.
17. Berry, P. and Kaufman, H., "Development of a Digital Adaptive Optimal Linear Regulator Flight Controller," IFAC/75 Congress, Boston, Aug. 1975.
18. Kaufman, H., "Aircraft Parameter Identification Using Kalman Filtering," Proc. of the 1969 National Electronics Conference, Chicago, pp. 85-89.
19. Armstrong, E. S., "Digital Explicit Model Following with Unstable Model Dynamics," AIAA Mechanics and Control of Flight Conf., Anaheim, CA., Aug. 1974, paper No. 74-888.
20. Anderson, B.D.O. and Moore, J.B., Linear Optimal Control, Prentice Hall, 1971.
21. Hewer, G. A., "An Iterative Technique for the Computation of the Steady State Gains for the Discrete Optimal Regulator," IEEE Trans. Automatic Control, Vol. AC-16, Aug. 1971, pp. 382-384.
22. Kleinman, D. L., "Stabilizing a Discrete, Constant, Linear Systems with Application to Iterative Methods for Solving the Riccati Equation, IEEE Trans. Automatic Control, Vol. AC-19, June 1974, pp. 252-254.
23. Alag, G.; Rondot, J.; Kaufman, H.; "Digital Adaptive Model Following Control," IEEE SMC Conf., Boston, Nov. 1973, pp. 104-106.
24. Chang, S.S.L. and Peng, T.K.C., "Adaptive Guaranteed Cost Control of Systems with Uncertain Parameters," IEEE Trans. on Automatic Control, Vol. AC.17, Aug. 1972, pp. 474-483.
25. Osburn, P. V., Whitaker, H. P. and Kezer, A., "New Developments in the Design of Model Reference Adaptive Control Systems, IAS Paper No. 61-34, IAS 29th Annual Meeting, New York, NY, Jan. 23-25, 1961.
26. Taylor, L. W., and Smith, J. W., "An Analysis of the Limit-Cycle and Structural Resonance Characteristics of the X-15 Stability Augmentation System," NASA TN D-4287, Dec. 1967.

27. Anderson, W. N.; Kleindorfer, G. B.; Kleindorfer, P. R.; and Woodroffe, M. B., "Consistent Estimates of the Parameters for Linear Systems", Annals of Math Stat., Vol. 40, No. 6, 1969, pp. 2064-2075.
28. Young, P.C., "Applying Parameter Estimation to Dynamic Systems-Part I", Control Engineering, Oct. 1969, Vol. 16, No. 10, 119-125.
29. Mendel, J. M., Discrete Techniques of Parameter Estimation, Marcel Dekker, Inc., New York, 1973.
30. Kotob, S., "Online Discrete System Identification with Applications to Adaptive Control," Doctoral Dissertation, Electrical and Systems Engineering Dept., Rensselaer Polytechnic Institute, Troy, NY, September 1975.
31. Kaufman, H., "Kalman Filtering and Quasilinearization: A Comparative Discussion of Two Procedures for Parameter Estimation," IEEE International Conference on Information Theory, Noordwijk, Netherlands, June 1970.
32. Kotob, S. and Kaufman, H., "Digital System Identification and Its Application to Digital Flight Control," 4th Symp. on Nonlinear Estimation, San Diego, Sept. 1973, pp. 175-178.
33. Mehra, R. K., "Optimal Inputs for Linear Systems Identification," IEEE Trans. on Automatic Control, June 1974, pp. 192-200.
34. Mehra, R. K., "Optimal Input Signals for Parameter Estimation in Dynamic Signals - Survey and New Results," IEEE Trans. on Automatic Control, Dec. 1974, pp. 753-758.
35. Wooley, C. T., "A Real Time Hybrid Simulation of a Digital Fly-by-Wire Aircraft," 1974 Summer Computer Simulation Conf., Houston, July 1974.

Appendix A

A.1 Aircraft Continuous Lateral Matrices

	F	G
FC1	$\begin{bmatrix} -3.598 & .1968 & -35.18 & 0 \\ -.0377 & -.3576 & 5.884 & 0 \\ .9688 & -.9957 & -.2163 & .0733 \\ .9947 & .1027 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 14.65 & 6.538 \\ .2179 & -3.087 \\ -.0054 & .0516 \\ 0 & 0 \end{bmatrix}$
FC2	$\begin{bmatrix} -10.22 & -.1416 & -147.8 & 0 \\ .0671 & -.9610 & 29.43 & 0 \\ -.0101 & -.9958 & -.5613 & .0309 \\ .9997 & .0245 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 77.86 & 42.61 \\ .9165 & -14.40 \\ -.0247 & .0864 \\ 0 & 0 \end{bmatrix}$
FC3	$\begin{bmatrix} -8.675 & -.1313 & -155.2 & 0 \\ .1078 & -.9961 & 30.57 & 0 \\ -.0145 & -.9967 & -.5710 & .0274 \\ .9998 & .0207 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 54.30 & 43.22 \\ .7347 & -9.961 \\ -.0176 & .0538 \\ 0 & 0 \end{bmatrix}$
FC4	$\begin{bmatrix} -1.377 & .2230 & -33.13 & 0 \\ -.0037 & -.1955 & 6.710 & 0 \\ .1152 & -.9992 & -.1074 & .0302 \\ .9888 & .1494 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 11.63 & 4.435 \\ .2086 & -1.761 \\ -.0014 & .0107 \\ 0 & 0 \end{bmatrix}$
FC5	$\begin{bmatrix} -1.525 & .0678 & -30.02 & 0 \\ -.0116 & -.1502 & 5.159 & 0 \\ .0698 & -.9992 & -.0903 & .0350 \\ .9945 & .1044 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 11.51 & 5.241 \\ .1894 & -1.968 \\ -.0030 & .0135 \\ 0 & 0 \end{bmatrix}$
FC6	$\begin{bmatrix} -4.033 & .0630 & -53.69 & 0 \\ -.0158 & -.3688 & 8.814 & 0 \\ .0236 & -.9971 & -.2333 & .0463 \\ .9983 & .0592 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 26.04 & 11.96 \\ .3293 & -4.823 \\ -.0100 & .0528 \\ 0 & 0 \end{bmatrix}$
MODEL	$\begin{bmatrix} -10.0 & 0 & +10.0 & 0 \\ 0 & -.7 & 9. & 0 \\ 0 & -1. & -.7 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 20 & 2.8 \\ 0 & -3.13 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

A.2 Aircraft Continuous Longitudinal Matrices

	F	G
FC	$\begin{bmatrix} -.473 & -.00039 & -2.03 & 0 \\ 0 & -.0287 & -19.5 & -32.2 \\ 1. & -.00059 & 0.803 & 0 \\ 1. & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -5.05 & 0 \\ -2.57 & 12.7 \\ -.115 & 0 \\ 0 & 0 \end{bmatrix}$
FC2	$\begin{bmatrix} -1.18 & -.00039 & -20.9 & 0 \\ 0 & -.0249 & -13.8 & -32.2 \\ 1. & -.00007 & -2.67 & 0 \\ 1. & 0. & 0. & 0. \end{bmatrix}$	$\begin{bmatrix} -33.1 & 0. \\ 1.59 & 11.4 \\ -.259 & 0. \\ 0. & 0. \end{bmatrix}$
FC3	$\begin{bmatrix} -.415 & .00062 & -43.0 & 0 \\ 0 & -.00199 & -16.7 & -32.2 \\ 1 & .00002 & -1.59 & 0. \\ 1 & 0. & 0. & 0. \end{bmatrix}$	$\begin{bmatrix} -25.3 & 0 \\ -.881 & 9.95 \\ -.165 & 0. \\ 0. & 0. \end{bmatrix}$
FC4	$\begin{bmatrix} -.280 & .0008 & 021.7 & 0. \\ 0. & -.00126 & -23.0 & -32.2 \\ 1. & 0. & -.776 & 0. \\ 1. & 0. & 0. & 0. \end{bmatrix}$	$\begin{bmatrix} -13.5 & 0. \\ .856 & 2.21 \\ 0.0867 & 0. \\ 0. & 0. \end{bmatrix}$
FC5	$\begin{bmatrix} -.402 & -.00075 & -5.86 & 0 \\ 0 & -.0113 & -23.4 & -32.2 \\ 1. & -.00008 & -.653 & 0. \\ 1. & 0. & 0 & 0. \end{bmatrix}$	$\begin{bmatrix} -9.29 & 0 \\ -1.05 & 2.13 \\ -.0729 & 0. \\ 0. & 0. \end{bmatrix}$
FC6	$\begin{bmatrix} -.551 & 0. & -5.02 & 0. \\ 0. & 0. & 0. & -32.2 \\ 1. & 0. & -.734 & 0. \\ 1. & 0. & 0. & 0. \end{bmatrix}$	$\begin{bmatrix} -9.49 & 0. \\ 0. & 9.18 \\ -.103 & 0. \\ 0. & 0. \end{bmatrix}$

Appendix B

Noise Characteristics

Variable	Rms	Bandwidth
p, Roll Rate	2 Deg/Sec	2 Hz
q, Pitch Rate	.5 Deg/Sec	2 Hz
r, Yaw Rate	.5 Deg/Sec	2 Hz
v, Velocity	2. Ft/Sec	1. Hz
β , Sideslip	.3 Deg	30 Hz
α , Angle-of-Attack	.3 Deg	30 Hz
ϕ , Bank Angle	.2 Deg	1. Hz
θ , Pitch Angle	.2 Deg	1. Hz

Appendix C

Flight Trajectory Used For Evaluation

<u>FC</u>	<u>Time of encounter (seconds)</u>
1	0
2	30
3	35
4	80
5	85
6	120